# 4-6 Related Rates

- Compare two or more variables with respect to time.

$$\frac{dr}{dt} = \frac{\Delta r}{\Delta t}$$

How fast is the radius changing?

$$\frac{dV}{dt} = \frac{\Delta V}{\Delta t}$$

How fast is the volume changing?

$$\frac{dh}{dt} = \frac{\Delta h}{\Delta t}$$

How fast is the height changing?

Ex 1) $y = x^2 + 3x$

Find dy/dt when x = 3 and dx/dt = 2.

$$1 \cdot \frac{dy}{dt} = 2x \cdot \frac{dx}{dt} + 3 \cdot \frac{dx}{dt}$$

$$\frac{dy}{dt} = 2(3) \cdot 2 + 3 \cdot 2$$

$$\frac{dy}{dt} = 12 + 6 = \boxed{18}$$

Ex 2) $x^2 + y^2 = 25$

Find dy/dt when x = 3, y = 4 and dx/dt = 8.

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

$$2 \cdot 3 \cdot 8 + 2(4) \cdot \frac{dy}{dt} = 0$$

$$48 + 8\frac{dy}{dt} = 0$$

$$8\frac{dy}{dt} = -48$$

$$\frac{dy}{dt} = -6$$

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1. Label all variables.

2. Write an equation relating the variables.

3. Differentiate explicitly with respect to t.

4. Substitute into the derivative equation.

5. Solve

Ex 3) Air is being released from a spherical balloon at 3 $in^3/min$. What is the rate of a change for the radius, when r = 2 in?

$\frac{dr}{dt} = ?$ $\frac{dV}{dt} = -3 in^3/min$ $r = 2$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3} \cdot \pi \cdot 3 \cdot r^2 \cdot \frac{dr}{dt}$

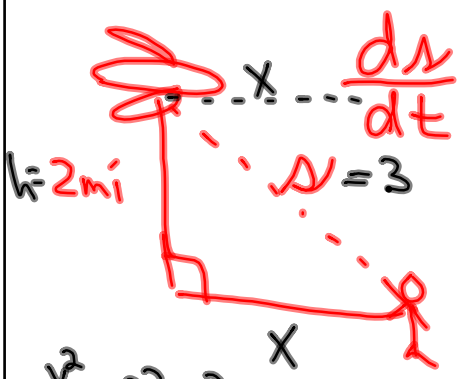
$-3 = 4\pi \cdot (2)^2 \frac{dr}{dt}$

$-3 = 16\pi \cdot \frac{dr}{dt}$

$\frac{-3}{16\pi} = \frac{dr}{dt}$

$\frac{-3}{16\pi}$ in/min

Ex 4) An airplane is flying at an altitude of 2 mi. If the distance, s, from the plane to a person on the ground is decreasing at 300 mph, what is the speed of the plane when s is 3 miles?




$\frac{ds}{dt} = -300 \text{ mph}$

$s = 3 \text{ mi}$ $\quad \frac{dh}{dt} = 0$

$\frac{dx}{dt} = ?$

$x^2 + h^2 = s^2$

$x^2 + 2^2 = 3^2$

$x^2 + 4 = 9$

$x^2 = 5$

$x = \pm\sqrt{5}$

$2x \cdot \frac{dx}{dt} + 2 \cdot h \cdot \frac{dh}{dt} = 2s \cdot \frac{ds}{dt}$

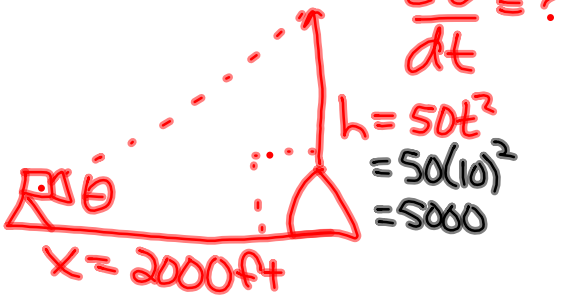
$2\sqrt{5} \frac{dx}{dt} + 2 \cdot 2 \cdot 0 = 2 \cdot 3 \cdot -300$

$2\sqrt{5} \frac{dx}{dt} = -1800$

$\frac{dx}{dt} = \frac{-1800}{2\sqrt{5}} = \frac{-900}{\sqrt{5}} = -402.5 \text{ mph}$

Ex 5) A TV camera 2000 ft from the launch pad films the lift off for a shuttle. The shuttle is rising such that $h = 50t^2$.

Find the rate of change for the camera's angle of elevation at t = 10 sec.




$\frac{d\theta}{dt} = ?$

$h = 50t^2$

$= 50(10)^2$

$= 5000$

$\frac{dx}{dt} = 0$

$\tan\theta = \frac{h}{x}$

$\tan\theta = \frac{5000}{2000}$

$\tan^{-1}\left(\frac{5}{2}\right) = \theta$

$68.19^\circ = \theta$

$\frac{dh}{dt} = 100t$

$= 100 \cdot 10$

$= 1000$

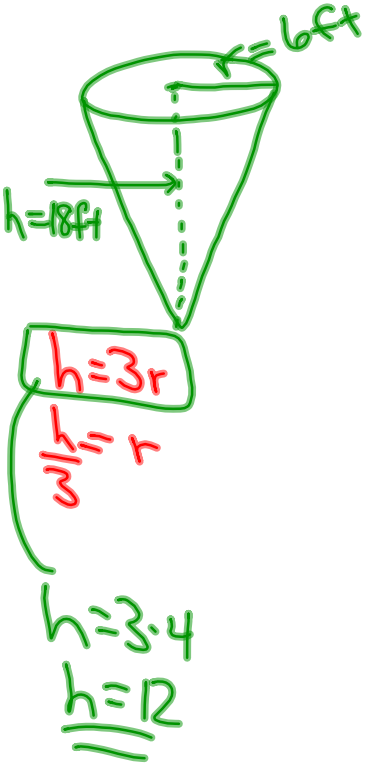
$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{x \cdot \frac{dh}{dt} - h \cdot \frac{dx}{dt}}{x^2}$

$\sec^2(68.19)\frac{d\theta}{dt} = \frac{2000 \cdot 1000 - 5000 \cdot 0}{2000^2}$

$\sec^2(68.19)\cdot\frac{d\theta}{dt} = \frac{1}{2}$

$\frac{d\theta}{dt} = \frac{.5}{\sec^2(68.19)} = .069^\circ/\text{sec}$

Ex 6) A water tank in the shape of a cone with radius = 6 ft and height = 18 ft, is leaking water at 2 $ft^3/hr$. How fast is the height changing when the radius = 4 ft?




$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$

$V=\frac{\pi h^3}{27}=\frac{\pi}{27}\cdot h^3$

$\frac{dV}{dt}=\frac{\pi}{9}\cdot h^2\cdot\frac{dh}{dt}$

$-2=\frac{\pi}{9}\cdot 12^2\cdot\frac{dh}{dt}$

$-2=\frac{144\pi}{9}\cdot\frac{dh}{dt}$

$\frac{dh}{dt}=\frac{-2}{16\pi}=-.0398\ ft/hr$

$r=4$

$h=12$

$\frac{dV}{dt}=-2\ ft^3/hr$

$\frac{dh}{dt}=?$