

Related Rates

4.6

Solve each related rate problem.

- 1) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 4 cm/min. How fast is the area of the pool increasing when the radius is 5 cm?

$$\frac{dA}{dt}$$

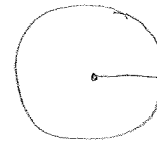
$$A = \pi r^2$$

$$r = 5$$

$$\frac{dr}{dt} = 4$$

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(5)(4) = \boxed{40\pi \text{ cm}^2/\text{min}}$$



- 2) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The area of the spill increases at a rate of $9\pi \text{ m}^2/\text{min}$. How fast is the radius of the spill increasing when the radius is 10 m?

$$\frac{dA}{dt} = 9\pi \text{ m}^2/\text{min}$$

$$\frac{dr}{dt} = ?$$

$$r = 10$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$9\pi = 2\pi \cdot 10 \cdot \frac{dr}{dt}$$

$$9\pi = 20\pi \cdot \frac{dr}{dt}$$

$$\frac{9 \text{ m}^2/\text{min}}{20} = \frac{dr}{dt}$$

$$\frac{9\pi}{20\pi} = \frac{dr}{dt}$$



- 3) A conical paper cup is 10 cm tall with a radius of 10 cm. The cup is being filled with water so that the water level rises at a rate of 2 cm/sec. At what rate is water being poured into the cup when the water level is 8 cm?

$$h = 10, r = 10 \quad \frac{dh}{dt} = 2 \text{ cm/sec}$$

$$h = 8$$

$$\frac{dV}{dt} = ?$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{\pi h^3}{3}$$

$$\frac{dV}{dt} = \pi (8)^2 \cdot 2 = \boxed{128\pi \text{ cm}^3/\text{sec}}$$

$$\frac{r}{h} = \frac{10}{10}$$

$$\frac{r}{h} = \frac{10}{8}$$

$$r = h$$

$$\frac{dV}{dt} = \pi \cdot h^2 \cdot \frac{dh}{dt}$$

