AP Calculus 4.6 AP Free Response #5 **Related Rates** Key

$A = l \cdot w$

$A = 200\text{m}^2$ l, w $\frac{dl}{dt} = 4\text{m/s}$

1. **AP 1977 – AB 6**

A rectangle has a constant area of 200 square meters and its length L is increasing at a rate of 4 meters per second.

$-dw/dt = .5\text{m/s}$

a. Find the width W at the instant the width is decreasing at a rate if 0.5 meters per second.

b. At what rate is the diagonal D of the rectangle changing at the instant the width W is 10 meters?

a) $A = l \cdot w$

$200 = l \cdot w \rightarrow w = \frac{200}{l}$

$0 = l \cdot \frac{dw}{dt} + w \cdot \frac{dl}{dt}$

$0 = l \cdot -.5 + \frac{200}{l} \cdot 4$

$\left[0 = -.5l + \frac{800}{l}\right] l$

$0 = -.5l^2 + 800$

$-800 = -.5l^2$

$1600 = l^2$

$l = \sqrt{1600}$

$l = 40$

$W = \frac{200}{40} = \boxed{5\text{m}}$

b) $\frac{dd}{dt} = ?$

$l = \frac{200}{10} = 20$

$d^2 = 20^2 + 10^2$

$d = 22.36$

$w = 10$, $l = 20$

$l^2 + w^2 = d^2$

$2l \cdot \frac{dl}{dt} + 2w \cdot \frac{dw}{dt} = 2 \cdot d \cdot \frac{dd}{dt}$

$2 \cdot 20 \cdot 4 + 2 \cdot 10 \cdot -2 = 2(22.36)\frac{dd}{dt}$

$160 - 40 = 44.72 \cdot \frac{dd}{dt}$

$\frac{120}{44.72} = \frac{dd}{dt} = \boxed{2.68 \text{ m/s}}$

$0 = 20\left(\frac{dw}{dt}\right) + 10(4)$

$-40 = 20 \cdot \frac{dw}{dt}$

$-2 = \frac{dw}{dt}$

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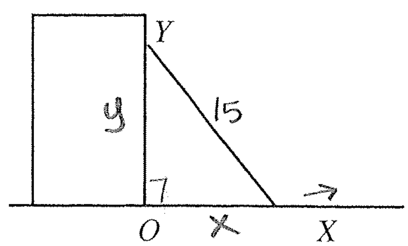
2. **AP 1982 – AB 4**

A ladder 15 feet long is leaning against a building so that end *X* is on level ground and end *Y* is on the wall of as shown in the figure. *X* is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second.

$y^2 + 9^2 = 15^2$

$y = 12$

$\frac{dx}{dt} = \frac{1}{2}\text{ft/sec}$



a. Find the rate in feet per second at which the length of *OY* is changing when *X* is 9 feet from the building.

$\frac{dy}{dt} = ?$ $x = 9$

b. Find the rate of change in square feet per second of the area of the triangle *XOY* when *X* is 9 feet from the building.

a) $x^2 + y^2 = 15$

$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

$2 \cdot 9 \cdot \frac{1}{2} + 2 \cdot 12 \cdot \frac{dy}{dt} = 0$

$9 + 24\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-9}{24} = -.375\text{ft/sec}$

b. $\frac{dA}{dt} = ?$

$A = \frac{1}{2} \cdot x \cdot y$

$\frac{dA}{dt} = \frac{1}{2}x \cdot \frac{dy}{dt} + \frac{1}{2}\frac{dx}{dt} \cdot y$

$\frac{dA}{dt} = \frac{1}{2} \cdot 9(-.375) + \frac{1}{2}\left(\frac{1}{2}\right)(12)$

$\frac{dA}{dt} = \boxed{+1.3125 \frac{\text{ft}^2}{\text{sec}}}$

3. **AP 1985 – AB 5, BC 2**

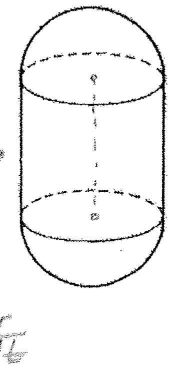
The balloon shown below is in the shape of a cylinder with a hemispherical ends of the same radius as the cylinder. The balloon is being inflated at the rate of 216π cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters, the volume of the balloon is 144π cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute. (The volume of a cylinder with radius r and height h is $\pi r^2 h$, and the volume of a sphere with radius r is $\frac{4}{3}\pi r^3$.)

$\frac{dV}{dt} = 216\pi \frac{cm^3}{min}$

$r = 3$

$V = 144\pi\ cm^3$

$\frac{dr}{dt} = 2cm/min$



a. At this instant, what is the height of the cylinder? $h = ?$

b. At this instant, how fast is the height of the cylinder increasing? $\frac{dh}{dt} = ?$

a) $144\pi = \pi r^2 h + \frac{4}{3}\pi r^3$

$144\pi = \pi \cdot 9h + \frac{4}{3}\pi \cdot 27$

$144\pi = 9\pi h + 36\pi$

$108\pi = 9\pi \cdot h$

$\frac{108\pi}{9\pi} = h \quad \boxed{h = 12cm}$

b) $V = \pi r^2 h + \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 2\pi r \cdot \frac{dr}{dt} \cdot h + \pi r^2 \cdot \frac{dh}{dt} + \frac{4}{3} \cdot \pi \cdot 3r^2 \cdot \frac{dr}{dt}$

$216\pi = 2\pi \cdot 3 \cdot 2 \cdot 12 + \pi \cdot 9 \cdot \frac{dh}{dt} + 4\pi \cdot 9 \cdot 2$

$216\pi = 144\pi + 9\pi \cdot \frac{dh}{dt} + 72\pi$

$0 = 9\pi \cdot \frac{dh}{dt}$

$0 = \frac{dh}{dt} = \boxed{0\ cm/min}$

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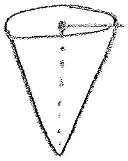
5. **AP 1984 – AB 5**



$dV/dt = 28\pi$ units/sec

The volume V of a cone $\left(V = \frac{1}{3}\pi r^2 h\right)$ is increasing at the rate of 28π cubic units per second. At the instant when the radius r of the cone is 3 units, its volume is 12π cubic units and the radius is increasing at $\frac{1}{2}$ unit per second. $r = 3$ $\quad \frac{dr}{dt} = \frac{1}{2}$ unit/sec $\quad V = 12\pi$

$12\pi = \frac{1}{3}\pi(3)^2 h$

$h = \frac{12\pi}{3\pi} = 4$

a. At the instant when the radius of the cone is 3 units, what is the rate of change of the area of its base?

b. At the instant when the radius of the cone is 3 units, what is the rate of change of its height h? $dh/dt = ?$

c. At the instant when the radius of the cone is 3 units, what is the instantaneous rate of change of the area of its base with respect to its height h? $\rightarrow \frac{dA}{dh} = ?$

A] $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(3) \cdot \frac{1}{2}$

$= \boxed{3\pi\ units^2/sec}$ or $9.424\ u^2/sec$

B] $V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{2}{3}\pi r \cdot h \cdot \frac{dr}{dt} + \frac{1}{3}\pi r^2 \cdot \frac{dh}{dt}$

$28\pi = \frac{2}{3}\pi \cdot 3 \cdot 4 \cdot \frac{1}{2} + \frac{1}{3}\pi \cdot 9 \cdot \frac{dh}{dt}$

$28\pi = 4\pi + 3\pi \frac{dh}{dt}$

$\frac{28\pi - 4\pi}{3\pi} = \frac{dh}{dt}$

$\boxed{8\ units/sec} = dh/dt$

C) $A = \pi r^2$

$\frac{dA}{dh} = 2\pi r \cdot \frac{dr}{dh}$

$\frac{dr}{dh} = \frac{dr}{dt} \cdot \frac{dt}{dh} = \frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16}$

$\frac{dA}{dh} = 2\pi \cdot 3 \cdot \frac{1}{16}$

$= \frac{6\pi}{16} = 1.178\ u^2/u$

or $\boxed{1.178\ u}$