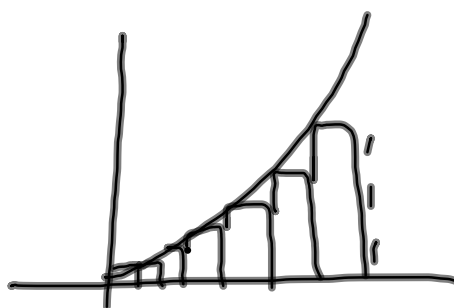
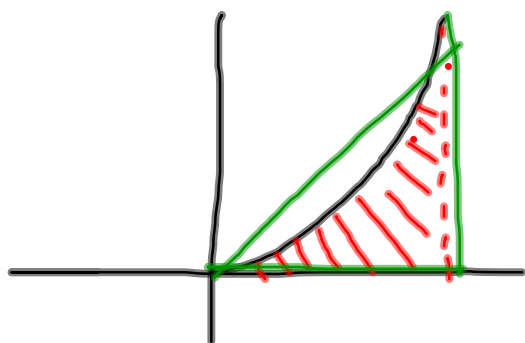


5.1 Estimating with Finite Sums

$$\int_a^b f(x) dx$$

↳ sum

→ Find the area
between the curve $f(x)$
and the x-axis
between $[a, b]$



RAM = Rectangle Approximation Method

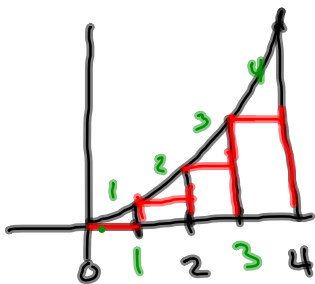
LRAM - **Left** Rectangle Approximation Method

RRAM - **Right** Rectangle Approximation Method

MRAM - **Midpoint** Rectangle Approximation Method

Ex 1) Estimate the area under $y = x^2$ $[0, 4]$ using 4 rectangles.

LRAM



$$A = bh$$

$$\#1 = 1 \cdot 0 = 0$$

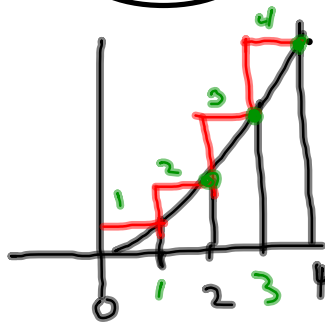
$$\#2 = 1 \cdot 1 = 1$$

$$\#3 = 1 \cdot 4 = 4$$

$$\#4 = 1 \cdot 9 = 9$$

$$\underline{14h^2}$$

RHAM



$$A = bh$$

$$\#1 = 1 \cdot 1 = 1$$

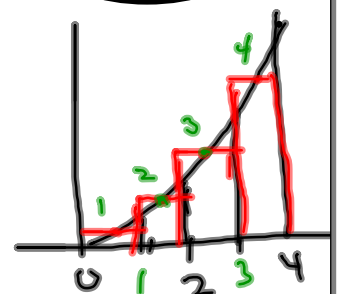
$$\#2 = 1 \cdot 4 = 4$$

$$\#3 = 1 \cdot 9 = 9$$

$$\#4 = 1 \cdot 16 = 16$$

$$\underline{30h^2}$$

MRAM



$$A = bh$$

$$\#1 = 1 \cdot 0.25 = 0.25$$

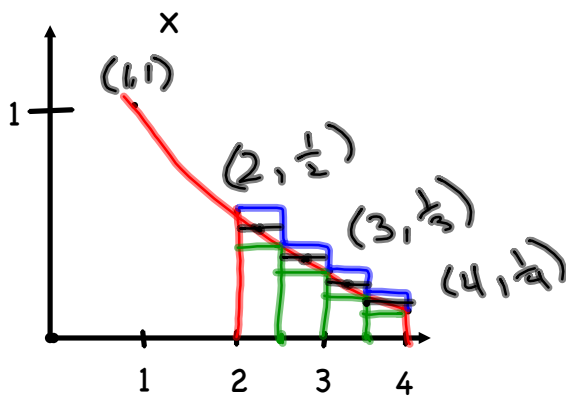
$$\#2 = 1 \cdot 1.25 = 1.5625$$

$$\#3 = 1 \cdot 2.25 = 5.0625$$

$$\#4 = 1 \cdot 3.25 = 10.5625$$

$$\underline{21h^2}$$

Ex 2) $y = \frac{1}{x}$ Estimate the area in the interval $[2, 4]$ using 4 rectangles.



a b

$$\text{base} = \frac{b - a}{\# \text{ of rectangles}}$$

$$= \frac{4 - 2}{4} = \frac{2}{4} = \frac{1}{2}$$

LRAM

$$A = bh$$

$$\begin{aligned} \#1 &= (.5)(\frac{1}{2}) \\ \#2 &= (.5)(\frac{1}{2.5}) \\ \#3 &= (.5)(\frac{1}{3}) \\ \#4 &= (.5)(\frac{1}{3.5}) \end{aligned} \quad \left. \vphantom{\begin{aligned} \#1 \\ \#2 \\ \#3 \\ \#4 \end{aligned}} \right\} = .76u^2$$

RRAM

$$A = bh$$

$$\begin{aligned} \#1 &= (.5)(\frac{1}{2.5}) \\ \#2 &= (.5)(\frac{1}{3}) \\ \#3 &= (.5)(\frac{1}{3.5}) \\ \#4 &= (.5)(\frac{1}{4}) \end{aligned} \quad \left. \vphantom{\begin{aligned} \#1 \\ \#2 \\ \#3 \\ \#4 \end{aligned}} \right\} = .63u^2$$

MRAM

$$A = bh$$

$$\begin{aligned} \#1 &= (.5)(\frac{1}{2.25}) \\ \#2 &= (.5)(\frac{1}{2.75}) \\ \#3 &= (.5)(\frac{1}{3.25}) \\ \#4 &= (.5)(\frac{1}{3.75}) \end{aligned} \quad \left. \vphantom{\begin{aligned} \#1 \\ \#2 \\ \#3 \\ \#4 \end{aligned}} \right\} = .69u^2$$