

5.2 Definite Integrals

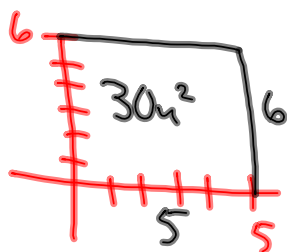
$$\int_a^b f(x) dx$$

Diagram illustrating the components of a definite integral:

- upper limit**: points to b
- integral sign**: points to \int
- lower limit**: points to a
- Integrand**: points to $f(x)$
- Variable of integration**: points to dx

Find the area between the curve and the x-axis on the interval $[a,b]$

Ex 1) $\int_0^5 6 dx = 30u^2$

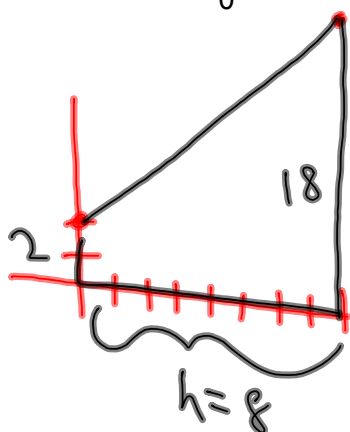


Note: $c = \text{constant}$

$$\int_a^b c \cdot dx = c(b-a)$$

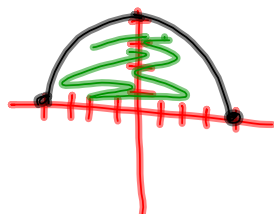
$$\begin{aligned}
 &= 6(5-0) \\
 &= 6 \cdot 5 \\
 &= \boxed{30u^2}
 \end{aligned}$$

$$\text{Ex 2) } \int_0^8 (2x + 2) dx = \boxed{80 \text{ u}^2}$$



$$\begin{aligned} A &= \frac{1}{2} \cdot 8(2 + 18) \\ &= 4 \cdot 20 \\ &= 80 \end{aligned}$$

$$\text{Ex 3) } \int_{-4}^4 \sqrt{16-x^2} dx$$

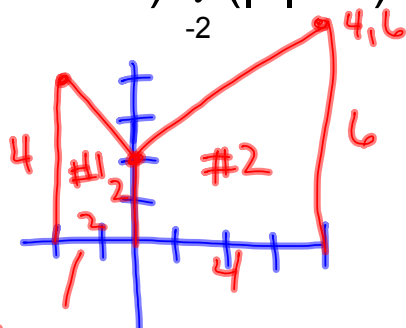


$$\begin{aligned} A &= \frac{1}{2} \pi r^2 \\ &= \frac{1}{2} \cdot \pi \cdot 4^2 = \boxed{8\pi \text{ u}^2} \end{aligned}$$

$$\begin{aligned} x^2 + y^2 &= r^2 \\ \sqrt{y^2} &= \sqrt{r^2 - x^2} \\ y &= \pm \sqrt{r^2 - x^2} \end{aligned}$$

$$\begin{aligned} y &= \sqrt{16-x^2} \\ y^2 &= 16-x^2 \\ x^2 + y^2 &= 16 \\ r &= 4 \end{aligned}$$

$$\text{Ex 4) } \int_{-2}^4 (|x| + 2) dx$$



$$\#1 \quad A = \frac{1}{2} \cdot 2(2+4) = \underline{6}$$

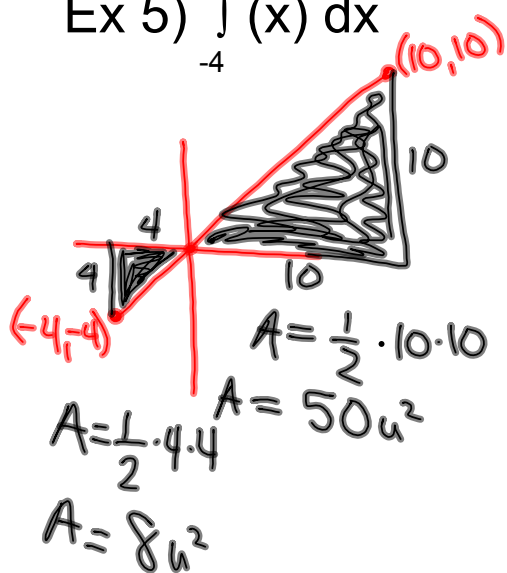
$$A = \frac{1}{2} \cdot 4(2+6) = \underline{16} \quad \text{--- } (22 \text{ u}^2)$$

$$\int_a^b f(x) dx \quad \text{Area above axis} - \text{Area below}$$

Area above axis is positive

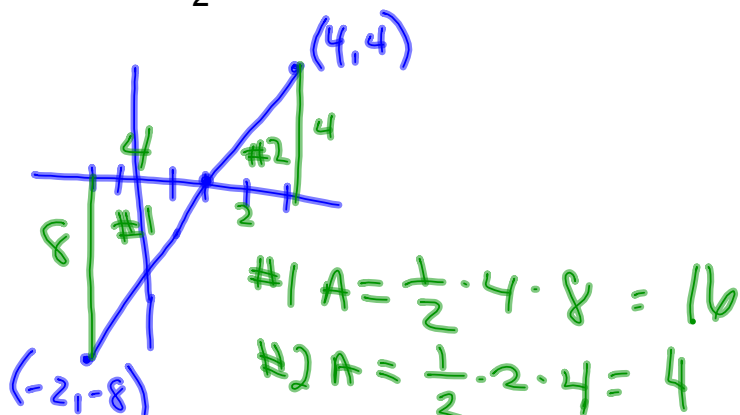
Area below axis is negative

Ex 5) $\int_{-4}^{10} (x) dx$



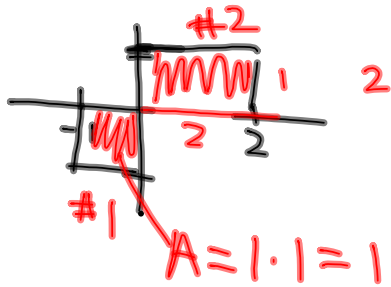
$$A = 50 - 8 = \boxed{42 \text{ u}^2}$$

Ex 6) $\int_{-2}^4 (2x - 4) dx$



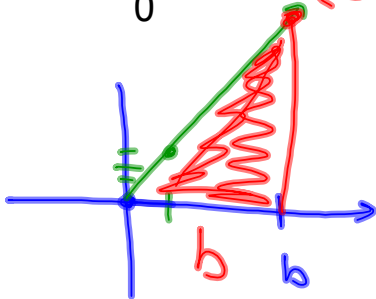
$$4 - 16 = \boxed{-12}$$

Ex 7) $\int_{-1}^2 (|x|/x) dx = 1$



$2 - 1 = 1$

Ex 8) $\int_0^b 3t dt$ $(b, 3b)$

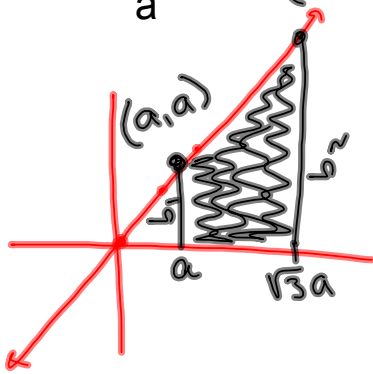


$A = \frac{1}{2} \cdot b \cdot h$
 $= \frac{1}{2} \cdot b \cdot 3b$
 $= \frac{3b^2}{2}$

$a > 0, b > 0$

Ex 9) $\int_a^{(\sqrt{3})a} x \, dx$

Way #1



$$A = \frac{1}{2} \cdot h(b_1 + b_2)$$

$$A = \frac{1}{2} \cdot (\sqrt{3}a - a)(a + \sqrt{3} \cdot a)$$

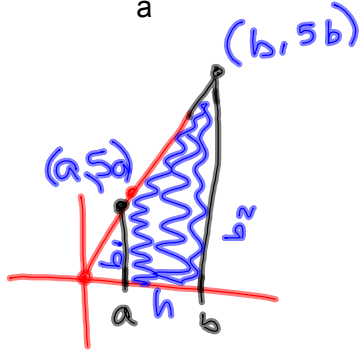
$$= \frac{1}{2}(3a^2 - a^2)$$

$$= \frac{1}{2}(2a^2) = a^2$$

Ex 10) $\int_a^{(\sqrt{3})a} x \, dx$

Way #2

Ex 11) $\int_a^b 5x \, dx$



$$\begin{aligned}
 A &= \frac{1}{2} h (b_1 + b_2) \\
 &= \frac{1}{2} \cdot (b-a) (5a + 5b) \\
 &= \frac{1}{2} (5ab + 5b^2 - 5a^2 - 5ab) \\
 &= \frac{1}{2} (5b^2 - 5a^2) \\
 &= \boxed{\frac{5}{2} b^2 - \frac{5}{2} a^2}
 \end{aligned}$$

Ex 12) $\int_a^b 2x \, dx$