Section 5.3 Exercises

Day 2

1. (a) $\int_2^2 g(x)\,dx = 0$

(b) $\int_5^1 g(x)\,dx = -\int_1^5 g(x)\,dx = -8$

(c) $\int_1^2 3f(x)\,dx = 3\int_1^2 f(x)\,dx = 3(-4) = -12$

(d) $\int_2^5 f(x)\,dx = \int_2^1 f(x)\,dx + \int_1^5 f(x)\,dx$

$= -\int_1^2 f(x)\,dx + \int_1^5 f(x)\,dx$

$= 4 + 6 = 10$

(e) $\int_1^5 [f(x) - g(x)]\,dx = \int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx$

$= 6 - 8 = -2$

(f) $\int_1^5 [4f(x) - g(x)]\,dx = \int_1^5 4f(x)\,dx - \int_1^5 g(x)\,dx$

$= 4\int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx$

$= 4(6) - 8 = 16$

3. (a) $\int_1^2 f(u)\,du = 5$

(b) $\int_1^2 \sqrt{3} f(z)\,dz = \sqrt{3}\int_1^2 f(z)\,dz = 5\sqrt{3}$

(c) $\int_2^1 f(t)\,dt = -\int_1^2 f(t)\,dt = -5$

(d) $\int_1^2 [-f(x)]\,dx = -\int_1^2 f(x)\,dx = -5$

5. (a) $\int_3^4 f(z)\,dz = \int_3^0 f(z)\,dz + \int_0^4 f(z)\,dz$

$= \int_0^3 f(z)\,dz + \int_0^4 f(z)\,dz$

$= -3 + 7 = 4$

(b) $\int_4^3 f(t)\,dt = \int_4^0 f(t)\,dt + \int_0^3 f(t)\,dt$

$= -\int_0^4 f(t)\,dt + \int_0^3 f(t)\,dt$

$= -7 + 3 = -4$

19. $\int_\pi^{2\pi} \sin x\,dx = -\cos(2\pi) + \cos(\pi)$

$= -2$

21. $\int_0^{\pi/1} e^x\,dx = e^1 - e^0 = e - 1$

23. $\int_1^4 2x\,dx = x^2\Big|_1^4 = 4^2 - 1^2 = 15$

11. An antiderivative of $x^2 - 1$ is $F(x) = \frac{1}{3}x^3 - x$.

$av = \frac{1}{\sqrt{3}}\int_0^{\sqrt{3}} (x^2 - 1)\,dx$

$= \frac{1}{\sqrt{3}}\left[F(\sqrt{3}) - F(0)\right]$

$= \frac{1}{\sqrt{3}}(0 - 0) = 0$

Find $x = c$ in $[0, \sqrt{3}]$ such that $c^2 - 1 = 0$

$c^2 = 1$

$c = \pm 1$

Since 1 is in $[0, \sqrt{3}]$, $x = 1$.

13. An antiderivative of $-3x^2 - 1$ is $F(x) = -x^3 - x$.

$av = \frac{1}{1}\int_0^1 (-3x^2 - 1)\,dx = F(1) - F(0) = -2$

Find $x = c$ in $[0, 1]$ such that $-3c^2 - 1 = -2$

$-3c^2 = -1$

$c^2 = \frac{1}{3}$

$c = \pm\frac{1}{\sqrt{3}}$

Since $\frac{1}{\sqrt{3}}$ is in $[0, 1]$, $x = \frac{1}{\sqrt{3}}$.

15. The region between the graph and the x-axis is a triangle of height 3 and base 6, so the area of the region is $\frac{1}{2}(3)(6) = 9$.

$av(f) = \frac{1}{6}\int_{-4}^{2} f(x)\,dx = \frac{9}{6} = \frac{3}{2}$.

17. There are equal areas above and below the x-axis.

$av(f) = \frac{1}{2\pi}\int_0^{2\pi} f(t)\,dt = \frac{1}{2\pi} \cdot 0 = 0$

25. $\int_{-2}^{6} 5\,dx = 5x\Big|_{-2}^{6} = 5(6) - 5(-2) = 40$

27. $\int_{-1}^{1} \frac{1}{1+x^2}\,dx = \tan^{-1}(1) - \tan^{-1}(-1) = \frac{\pi}{2}$

31. $av(f) = \frac{1}{\pi - 0}\int_0^\pi \sin x\,dx$

$= \frac{1}{\pi}(-\cos\pi - (-\cos 0)) = \frac{2}{\pi}$

35. $av(f) = \frac{1}{2-(-1)}\int_{-1}^{2} 3x^2 + 2x\,dx = \frac{1}{3}\left(x^3 + x^2\right)\Big|_{-1}^{2}$

$= 4$