5.4 Fundamental Theorem of Calculus

Fundamental Theorem of Calculus

$$\frac{d}{dx}\int_{a}^{x} f(t)\,dt = f(x)$$

any #

derivative of the Integral = original function

* The process of integration & differentiation are inverses of each other.

$$\frac{d}{dx}\left(F(x)-F(a)\right)=\frac{d}{dx}\left(F(x)-\#\right)$$

$$= f(x) + 0$$

Ex 1) $\frac{d}{dx}\int_7^x 2t\,dt = 2x$

$\frac{d}{dx}\left(\frac{\cancel{2}t^2}{\cancel{2}}\right)\Big|_7^x = \frac{d}{dx}\left(x^2 - 49\right)$

$= 2x + 0 = 2x$

Ex 2) $\frac{d}{dx}\int_{13}^{x} \cos t \, dt = \boxed{\cos x}$

$\frac{d}{dx}(\sin t)\Big|_{13}^{x} = \frac{d}{dx}(\sin x - \sin 13)$

$= \cos x + 0$

$= \boxed{\cos x}$

Ex 3) $\frac{d}{dx}\int_{1{,}000{,}007}^{x} n^3 \, dn = \boxed{x^3}$

$\frac{d}{dx}\left(\frac{n^4}{4}\right)\Big|_{1{,}000{,}007}^{x} = \frac{d}{dx}\left(\frac{x^4}{4} - \frac{1{,}000{,}007^4}{4}\right)$

$= x^3 + 0$

$= x^3$

Ex 4) $\frac{d}{dx}\int_x^7 2t\,dt = -\frac{d}{dx}\int_7^x 2t \cdot dt = \boxed{-2x}$

Ex 5) $\frac{d}{dx}\int_x^5 \sqrt{(u+1)}\,du =$

$-\frac{d}{dx}\int_5^x \sqrt{u+1}\,du = -\sqrt{x+1}$

Ex 6) $\frac{d}{dx}\int_3^{x^2} 2t\,dt = 2(x^2)\cdot 2x = \boxed{4x^3}$

$$\frac{d}{dx}\left(t^2\right)\Big|_3^{x^2} = \frac{d}{dx}\left(x^4 - 9\right)$$

$$= 4x^3 + 0 = 4x^3$$

Ex 7) $\frac{d}{dx}\int_4^{x^2}(\cos k)\,dk = 2x\cos(x^2)$

$$\frac{d}{dx}\left(\sin k\Big|_4^{x^2}\right) = \frac{d}{dx}\left(\sin(x^2) - \sin 4\right)$$

$$= 2x \cdot \cos x^2 - 0$$

Ex 8) $\frac{d}{dx}\int_x^{x^2}(2t^2+3)\,dt =$

$$\frac{d}{dx}\left(\frac{2}{3}t^3+3t\right)\Big|_x^{x^2} = \frac{d}{dx}\left(\frac{2}{3}(x^2)^3+3(x^2)-\left(\frac{2}{3}x^3+3x\right)\right)$$

$$= \frac{d}{dx}\left(\frac{2}{3}x^6+3x^2-\frac{2}{3}x^3-3x\right)$$

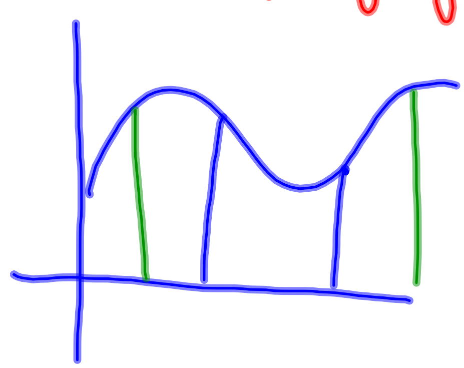
$$= \boxed{4x^5+6x-2x^2-3}$$

$$\frac{d}{dx}\left(\int_0^{x^2}(2t^2+3)\,dt-\int_0^{x}(2t^2+3)\,dt\right)$$

$$\left((2x^4+3)\cdot 2x-(2x^2+3)\right)$$

$$\boxed{4x^5+6x-2x^2-3}$$

$$\int_x^{x^2} = \int_0^{x^2} - \int_0^{x}$$



Ex 9) $\frac{d}{dx}\int_{x^2}^{x^3}(\sin t)\,dt =$

$$\frac{d}{dx}\left(-\cos t\right)\Big|_{x^2}^{x^3} = \frac{d}{dx}\left(-\cos(x^3) + \cos(x^2)\right)$$

$$= \sin(x^3)\cdot 3x^2 - \sin(x^2)\cdot 2x$$

$$= \boxed{3x^2\sin(x^3) - 2x\sin(x^2)}$$

$$\int_a^b v(t)\cdot dt = S(b) - S(a)$$