Section 5.4 Exercises

1. $\dfrac{dy}{dx}=\dfrac{d}{dx}\int_0^x\left(\sin^2 t\right)dt=\sin^2 x$

3. $\dfrac{dy}{dx}=\dfrac{d}{dx}\int_0^x\left(t^3-t\right)^5 dt=\left(x^3-x\right)^5$

Day3

7. $\dfrac{dy}{dx}=\dfrac{d}{dx}\int_7^x\dfrac{1+t}{1+t^2}dt=\dfrac{1+x}{1+x^2}$

9. $\dfrac{dy}{dx}=\dfrac{d}{dx}\int_0^{x^2}e^{t^3}dt=e^{x^2}\dfrac{du}{dx}=2x\,e^{x^2}$

15. $\dfrac{dy}{dx}=\dfrac{d}{dx}\int_{x^3}^5\dfrac{\cos t}{t^2-2}dt=-\dfrac{d}{dx}\int_5^{x^3}\dfrac{\cos t}{t^2-2}dt=\dfrac{\cos x^3}{x^6-2}\dfrac{du}{dx}$

$=-\dfrac{3x^2\cos x^3}{x^6+2}$

17. $\dfrac{dy}{dx}=\dfrac{d}{dx}\int_{\sqrt{x}}^0\sin\left(r^2\right)dr=-\dfrac{d}{dx}\int_0^{\sqrt{x}}\sin\left(r^2\right)dr$

$=-\sin x\dfrac{du}{dx}=-\dfrac{\sin x}{2\sqrt{x}}$

21. $y=\int_5^x\sin^3 t\,dt$

23. $\left|E_{S_{10n}}\right|=10^{-4}\left|E_{S_n}\right|$

Day2

5. $\dfrac{dy}{dx}=\dfrac{d}{dx}\int_0^x\left(\tan^3 u\right)dt=\tan^3 x$

11. $\dfrac{dy}{dx}=\dfrac{d}{dx}\int_2^{5x}\dfrac{\sqrt{1+u^2}}{u}du=\dfrac{\sqrt{1+25x^2}}{x}$

13. $\dfrac{dy}{dx}=\dfrac{d}{dx}\int_x^6\ln\left(1+t^2\right)dt=-\dfrac{d}{dx}\int_6^x\ln\left(1+t^2\right)dt$

$=\ln\left(1+x^2\right)$

27. $\int_{1/2}^3\left(2-\dfrac{1}{x}\right)dx=\left[2x-\ln|x|\right]_{1/2}^3$

$=(6-\ln 3)-\left(1-\ln\dfrac{1}{2}\right)$

$=5-\ln 3+\ln\dfrac{1}{2}$

$=5-\ln 3-\ln 2$

$=5-\ln 6\approx 3.208$

29. $\int_0^1(x^2+\sqrt{x})\,dx=\left[\dfrac{1}{3}x^3+\dfrac{2}{3}x^{3/2}\right]_0^1=\left(\dfrac{1}{3}+\dfrac{2}{3}\right)-(0+0)=1$

33. $\int_0^\pi\sin x\,dx=\left[-\cos x\right]_0^\pi=1-(-1)=2$

39. $\int_{-1}^1(r+1)^2\,dr=\left[\dfrac{1}{3}(r+1)^3\right]_{-1}^1=\dfrac{8}{3}-0=\dfrac{8}{3}$

45. First, find the area under the graph of $y=x^2$.

$\int_0^1 x^2\,dx=\left[\dfrac{1}{3}x^3\right]_0^1=\dfrac{1}{3}$

Next find the area under the graph of $y=2-x$.

$\int_1^2(2-x)dx=\left[2x-\dfrac{1}{2}x^2\right]_1^2=2-\dfrac{3}{2}=\dfrac{1}{2}$

Area of the shaded region $=\dfrac{1}{3}+\dfrac{1}{2}=\dfrac{5}{6}$

46. First find the area under the graph of $y=\sqrt{x}$.

$\int_0^1 x^{1/2}dx=\left[\dfrac{2}{3}x^{3/2}\right]_0^1=\dfrac{2}{3}$

Next find the area under the graph of $y=x^2$.

$\int_1^2 x^2\,dx=\left[\dfrac{1}{3}x^3\right]_1^2=\dfrac{8}{3}-\dfrac{1}{3}=\dfrac{7}{3}$

Area of the shaded region $=\dfrac{2}{3}+\dfrac{7}{3}=3$

47. First, find the area under the graph of $y=1+\cos x$.

$\int_0^\pi(1+\cos x)\,dx=\left[x+\sin x\right]_0^\pi=\pi$

The area of the rectangle is 2π.

Area of the shaded region $=2\pi-\pi=\pi$.

48. First, find the area of the region between $y=\sin x$ and the x-axis for $\left[\dfrac{\pi}{6},\dfrac{5\pi}{6}\right]$.

$\int_{\pi/6}^{5\pi/6}\sin x\,dx=\left[-\cos x\right]_{\pi/6}^{5\pi/6}=\dfrac{\sqrt{3}}{2}-\left(-\dfrac{\sqrt{3}}{2}\right)=\sqrt{3}$

The area of the rectangle is $\left(\sin\dfrac{\pi}{6}\right)\left(\dfrac{2\pi}{3}\right)=\dfrac{\pi}{3}$

Area of the shaded region $=\sqrt{3}-\dfrac{\pi}{3}$

51. $\dfrac{1}{2}\text{NINT}\left(\sqrt{\cos x},x,-1,1\right)\approx 0.914$

52. $\sqrt{8-2x^2}\ge 0$ between $x=-2$ and $x=2$

$\text{NINT}(\sqrt{8-2x^2},x,-2,2)\approx 8.886$

59. **(a)** $s'(3)=f(3)=0$ units/sec

(b) $s''(3)=f'(3)>0$ so acceleration is positive.

(c) $s(3)=\int_0^3 f(x)dx=\dfrac{1}{2}(-6)(3)=-9$ units

(d) $s(6)=\int_0^6 f(x)dx=\dfrac{1}{2}(-6)(3)+\dfrac{1}{2}(6)(3)=0$, so the particle passes through the origin at $t=6$ sec.

(e) $s''(t)=f'(t)=0$ at $t=7$ sec

(f) The particle is moving away from the origin in the negative direction on $(0,3)$ since $s(0)=0$ and $s'(t)<0$ on $(0,3)$. The particle is moving toward the origin on $(3,6)$ since $s'(t)>0$ on $(3,6)$ and $s(6)=0$. The particle moves away from the origin in the positive direction for $t>6$ since $s'(t)>0$.

(g) The particle is on the positive side since $s(9)=\int_0^9 f(x)dx>0$ (the area below the x-axis is smaller than the area above the x-axis).