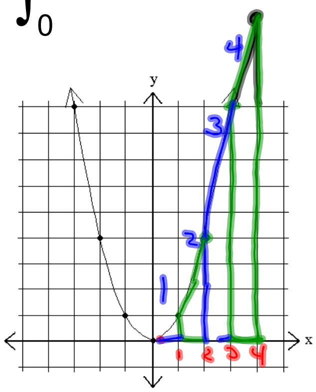
5.5 Trapezoidal Rule

Skip 13+15

Ex 1) Approximate the value of the integral.

$$\int_0^4 x^2\,dx$$



$A = \frac{1}{2}h(b_1+b_2)$

#1) $\frac{1}{2}\cdot 1(0+1) = \frac{1}{2}$

#2) $\frac{1}{2}\cdot 1(1+4) = 2\frac{1}{2}$

#3) $\frac{1}{2}\cdot 1(4+9) = 6\frac{1}{2}$

#4) $\frac{1}{2}\cdot 1(9+16) = 12\frac{1}{2}$

$\boxed{22}$

The Trapezoidal Rule $h = \frac{b-a}{n}$ n = Subintervals

To approximate $\int_a^b f(x)\,dx$, use $T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$

$\frac{LRAM + RRAM}{2}$ $\frac{1}{2}\cdot h$

Ex 2) Use the trapezoid rule with n = 4 to approximate the value of the integral.

$$\int_0^4 x^2\,dx$$

$T = \frac{1}{2}\cdot 1(0 + 2\cdot 1 + 2\cdot 4 + 2\cdot 9 + 16)$

$= \frac{1}{2}(44) = 22$

$h = \frac{4-0}{4} = 1$

Ex 3) Use the trapezoid rule with n = 4 to approximate the value of the integral.

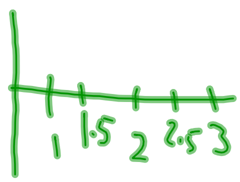
$$\int_1^3 \frac{1}{x}\,dx$$

$$T = \frac{1}{2}\cdot\frac{1}{2}\left(1 + 2\cdot\frac{2}{3} + 2\cdot\frac{1}{2} + 2\cdot\frac{2}{5} + \frac{1}{3}\right)$$

$$h = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$= \frac{1}{4}\left(1 + \frac{4}{3} + 1 + \frac{4}{5} + \frac{1}{3}\right)$$

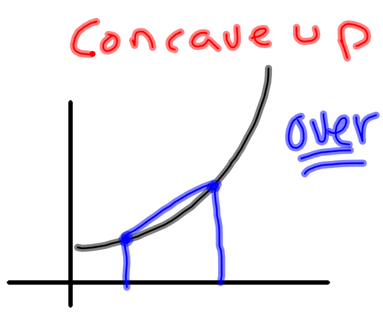
$$= \frac{1}{4}\left(\frac{67}{15}\right) = \boxed{1.111\overline{6}}$$

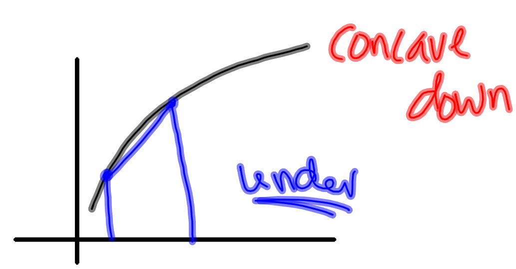


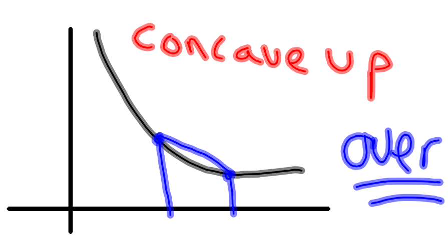
Check

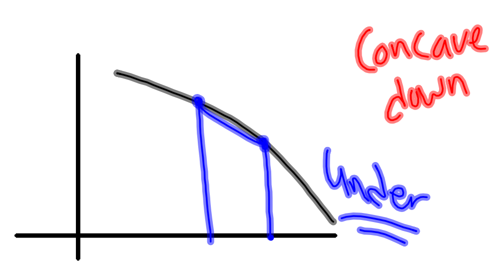
$$\ln x \Big|_1^3 = \ln 3 - \ln 1 = 1.0986$$

Ex 4) Use the concavity of the function to predict whether the approximation is an overestimate or an underestimate.









Ex 5) Use a trapezoid approximation to estimate distance traveled.

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#12

Time(s)	Speed (mph)
0	0
1	3
2	7
3	12
4	17
5	25
6	33
7	41
8	48

$\text{Time} \times \frac{\text{distance}}{\text{Time}}$

$T = \frac{1}{2}\cdot 1\left(0 + 2\cdot 3 + 2\cdot 7 + 2\cdot 12 + 2\cdot 17 + 2\cdot 25 + 2\cdot 33 + 2\cdot 41 + 48\right)$

$T = \frac{1}{2}(324)$

$= 162\ \cancel{\text{sec}} \times \frac{\text{miles}}{\cancel{\text{hr}}} \times \frac{1\ \cancel{\text{hr}}}{3600\ \cancel{\text{sec}}}$

$= \frac{162}{3600} = \underline{.045\ \text{miles}}$

$= \underline{237.6}\ \text{feet}$

$5280(.045)$