

Section 5.5 Exercises

1. (a) $f(x) = x, h = \frac{2-0}{4} = \frac{1}{2}$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2

$$T = \frac{1}{4} \left(0 + 2\left(\frac{1}{2}\right) + 2(1) + 2\left(\frac{3}{2}\right) + 2 \right) = 2$$

(b) $f'(x) = 1, f''(x) = 0$

The approximation is exact.

(c) $\int_0^2 x dx = \left[\frac{1}{2}x^2 \right]_0^2 = 2$

3. (a) $f(x) = x^3, h = \frac{2-0}{4} = \frac{1}{2}$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	0	$\frac{1}{8}$	1	$\frac{27}{8}$	8

$$T = \frac{1}{4} \left(0 + 2\left(\frac{1}{8}\right) + 2(1) + 2\left(\frac{27}{8}\right) + 8 \right) = 4.25$$

(b) $f'(x) = 3x^2, f''(x) = 6x > 0$ on $[0, 2]$

The approximation is an overestimate.

(c) $\int_0^2 x^3 dx = \left[\frac{1}{4}x^4 \right]_0^2 = 4$

7. $T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$

$$\int_0^6 f(x) dx \approx \frac{1}{2}(12 + 2(10) + 2(9) + 2(11) + 2(13) + 2(16) + 18) = 74$$

8. $T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$

$$\int_2^8 f(x) dx \approx \frac{1}{2}(16 + 2(19) + 2(17) + 2(14) + 2(13) + 2(16) + 20) = 97$$

10. (a) $\frac{200}{2}(0 + 2(520) + 2(800) + 2(1000) + \dots + 2(860) + 0)(20) = 26,360,000 \text{ ft}^3$

(b) You plan to start with 26,360 fish. You intend to have $(0.75)(26,360) = 19,770$ fish to be caught. Since

$$\frac{19,770}{20} = 988.5, \text{ the town can sell at most 988 licenses.}$$

13. (a) $\int_0^2 x dx = \left(\frac{1/2}{3} \right) \left(0 + 4\left(\frac{1}{2}\right) + 2(1) + 4\left(\frac{3}{2}\right) + 2 \right) = 2$

(b) $\int_0^2 x dx = \frac{x^2}{2} \Big|_0^2 = \frac{2^2}{2} + \frac{0^2}{2} = 2$

15. (a) $\int_0^2 x^3 dx = \left(\frac{1/2}{3} \right) \left(0^3 + 4\left(\frac{1}{2}\right)^3 + 2(1)^3 + 4\left(\frac{3}{2}\right)^3 + 2^3 \right) = 4$

(b) $\int_0^2 x^3 dx = \frac{x^4}{4} \Big|_0^2 = \frac{2^4}{4} - \frac{0^4}{4} = 4$