6.2 Antidifferentiation by Substitution Day 2

Definite Integral $\int_a^b f(x)\,dx$ today

height · base

Indefinite Integral $\int f(x)\,dx$

Ex 1) $\int_0^1 r\sqrt{1 - r^2}\,dr$

$u = 1 - r^2$

$\frac{du}{dr} = -2r$

$du = -2r \cdot dr$

$-\frac{1}{2}du = r \cdot dr$

$-\frac{1}{2}\int \sqrt{u} \cdot du = -\frac{1}{2}\int u^{1/2} \cdot du$

$= -\frac{1}{2} \cdot \frac{u^{3/2}}{3/2} + C = -\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3}(1-r^2)^{3/2} + C$

$= -\frac{1}{3}(1-r^2)^{3/2}\Big|_0^1 = -\frac{1}{3}(1-1^2)^{3/2} - \left(-\frac{1}{3}(1-0^2)^{3/2}\right)$

$= 0 + \frac{1}{3} \cdot 1 = \frac{1}{3}$

Ex 2) $\int_{-1}^{1} \frac{5r\,dr}{(4+r^2)^2}$

$u = 4+r^2$

$\frac{du}{dr} = 2r$

$du = 2\underline{r \cdot dr}$

$\frac{1}{2} du = r \cdot dr$

$5\int_{-1}^{1} \frac{r}{(4+r^2)^2} \cdot dr$

$5 \cdot \frac{1}{2} \int \frac{1}{u^2} du \leftarrow u^{-2}$

$= \frac{5}{2} \cdot \frac{u^{-1}}{-1} + C = -\frac{5}{2} \cdot \frac{1}{u} = -\frac{5}{2} \cdot \frac{1}{4+r^2}\Big|_{-1}^{1} = \frac{-5}{2} \cdot \frac{1}{5} - \frac{-5}{2} \cdot \frac{1}{5}$

$= -\frac{1}{2} + \frac{1}{2}$

$= 0$

OR $-\frac{5}{2} \cdot \frac{1}{u}\Big|_{5}^{5} = 0$

Ex 3) $\int_{2}^{5} \frac{dx}{2x-3}$

$u = 2x-3$

$du = 2 \cdot dx$

$\frac{1}{2} du = dx$

$= \frac{1}{2} \int \frac{1}{u} \cdot du$

$= \frac{1}{2} \ln u = \frac{1}{2} \ln(2x-3)\Big|_{2}^{5} = \frac{1}{2}\ln 7 - \frac{1}{2}\ln 1$

$= \frac{1}{2}\ln 7 - 0$

$= \frac{1}{2}\ln 7 \approx .97296$