Section 6.3 Exercises

1. $\int x \sin x \, dx$

$dv = \sin x \, dx \qquad v = \int \sin x \, dx = -\cos x$

$u = x \qquad du = dx$

$-x\cos x - \int -\cos x \, dx = -x\cos x + \sin x + C$

3. $\int 3t\, e^{2t} dt$

$dv = e^{2t} dt \qquad v = \int e^{2t} dt = \frac{e^{2t}}{2}$

$u = 3t \qquad du = 3\, dt$

$3t\frac{e^{2t}}{2} - \int 3\frac{e^{2t}}{2} dt = \frac{3}{2}te^{2t} - \frac{3}{4}e^{2t} + C$

5. $\int x^2 \cos x \, dx$

$dv = \cos x \, dx \qquad v = \int \cos x \, dx = \sin x$

$u = x^2 \qquad du = 2x \, dx$

$x^2 \sin x - \int 2x \sin x \, dx$

$dv = \sin x \, dx \qquad v = \int \sin x \, dx = -\cos x$

$u = 2x \qquad du = 2dx$

$x^2 \sin x + 2x\cos x - \int 2\cos x dx$

$= x^2 \sin x + 2x \cos x - 2\sin x + C$

7. $\int 3x^2 e^{2x} dx$

$dv = e^{2x} dx \qquad v = \int e^{2x} dx = \frac{e^{2x}}{2}$

$u = 3x^2 \qquad du = 6x$

$3x^2 \frac{e^{2x}}{2} - \int 6x\frac{e^{2x}}{2} dx = \frac{3}{2}x^2e^{2x} - \int 3x\, e^{2x} dx$

$dv = e^{2x} \qquad v = \int e^{2x} dx = \frac{e^{2x}}{2}$

$u = 3x \qquad du = 3dx$

$\frac{3}{2}x^2e^{2x} - \frac{3}{2}xe^{2x} - \int 3\frac{e^{2x}}{2} dx = \frac{3}{2}x^2e^{2x} - \frac{3}{2}xe^{2x} + \frac{3}{4}e^{2x} + C$

9. $\int y \ln y \, dy$

$dv = y\, dy \qquad v = \int y\, dy = \frac{y^2}{2}$

$u = \ln y \qquad du = \frac{1}{y} dy$

$\frac{1}{2}y^2 \ln y - \int \frac{y^2}{2}\frac{1}{y} dy = \frac{1}{2}y^2 \ln y - \frac{y^2}{4} + C$

11. $\int dy = \int \big((x+2)\sin x\big) dx$

$dv = \sin x \, dx \qquad v = \int \sin x \, dx = -\cos x$

$u = x + 2 \qquad du = dx$

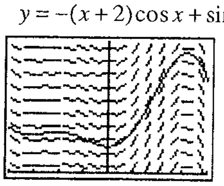
$-(x+2)\cos x - \int -\cos x \, dx = -(x+2)\cos x + \sin x + C$

$2 = -(0+2)\cos(0) + \sin(0) + C$

$2 = -2 + C$

$C = 4$

$y = -(x+2)\cos x + \sin x + 4$



[–4, 4] by [0, 10]

15. $\int dy = \int x\sqrt{x-1}\, dx$

$dv = (x-1)^{1/2} \qquad v = \int (x-1)^{1/2} dx = \frac{2}{3}(x-1)^{3/2}$

$u = x \qquad du = dx$

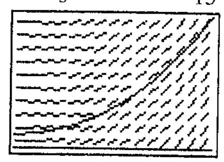
$\frac{2}{3}x\,(x-1)^{3/2} - \int \frac{2}{3}(x-1)^{3/2} dx$

$= \frac{2}{3}x(x-1)^{3/2} - \frac{4}{15}(x-1)^{5/2} + C$

$2 = \frac{2}{3}(1)\,(1-1)^{3/2} - \frac{4}{15}(1-1)^{5/2} + C$

$C = 2$

$y = \frac{2}{3}x(x-1)^{3/2} - \frac{4}{15}(x-1)^{5/2} + 2$



[1, 5] by [0, 20]

Day 2 on 6.3

17. $\int e^x \sin x dx$

$dv = e^x\, dx \qquad v = \int e^x\, dx = e^x$

$u = \sin x \qquad du = \cos x \, dx$

$e^x \sin x - \int e^x \cos x d$

$dv = e^x\, dx \qquad v = \int e^x\, dx = e^x$

$u = \cos x \qquad du = -\sin x \, dx$

$\int e^x \sin x dx = e^x \sin x - (e^x \cos x - \int -e^x \sin x dx)$

$\int e^x \sin x dx = \frac{e^x}{2}(\sin x - \cos x) + C$

19. $\int e^x \cos 2x \, dx$

$dv = \cos 2x \, dx \qquad v = \int \cos 2x \, dx = 2\sin 2x$

$u = e^x \qquad du = e^x dx$

$2e^x \sin 2x - \int 2\sin 2x \, e^x dx$

$dv = 2\sin 2x \qquad v = \int 2\sin 2x \, dx = -4\cos 2x$

$u = e^x \qquad du = e^x dx$

$\int e^x \cos 2x \, dx$

$= 2e^x \sin 2x - (-4e^x \cos 2x - \int -e^x dx \; 4\cos 2x \, dx)$

$\int e^x \cos 2x \, dx = \frac{e^x}{5}(2\sin 2x + \cos 2x) + C$

6.3 day 2

25. Use tabular integration with $f(x) = x^2$ and $g(x) = \sin 2x$.

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^2	(+)	$\sin 2x$
$2x$	(−)	$-\frac{1}{2}\cos 2x$
2	(+)	$-\frac{1}{4}\sin 2x$
0		$\frac{1}{8}\cos 2x$

$$\int x^2 \sin 2x\, dx = -\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C$$
$$= \left(\frac{1-2x^2}{4}\right)\cos 2x + \frac{x}{2}\sin 2x + C$$

$$\int_0^{\pi/2} x^2 \sin 2x\, dx = \left[\left(\frac{1-2x^2}{4}\right)\cos 2x + \frac{x}{2}\sin 2x\right]_0^{\pi/2}$$
$$= \left(\frac{1-2\left(\frac{\pi}{2}\right)^2}{4}\right)(-1) + 0 - \left(\frac{1}{4}\right)(1) - 0$$
$$= \frac{\pi^2}{8} - \frac{1}{2} \approx 0.734$$

Check: $\text{NINT}\left(x^2 \sin 2x, x, 0, \frac{\pi}{2}\right) \approx 0.734$

27. Let $u = e^{2x}$ $\quad dv = \cos 3x\, dx$

$du = 2e^{2x}\, dx$ $\quad v = \frac{1}{3}\sin 3x$

$$\int e^{2x}\cos 3x\, dx = (e^{2x})\left(\frac{1}{3}\sin 3x\right) - \int\left(\frac{1}{3}\sin 3x\right)(2e^{2x}\, dx)$$
$$= \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\int e^{2x}\sin 3x\, dx$$

Let $u = e^{2x}$ $\quad dv = \sin 3x\, dx$

$du = 2e^{2x}\, dx$ $\quad v = -\frac{1}{3}\cos 3x$

$$\int e^{2x}\cos 3x\, dx = \frac{1}{3}e^{2x}\sin 3x$$
$$-\frac{2}{3}\left[(e^{2x})\left(-\frac{1}{3}\cos 3x\right) - \int\left(-\frac{1}{3}\cos 3x\right)(2e^{2x}\, dx)\right]$$
$$= \frac{1}{9}e^{2x}(3\sin 3x + 2\cos 3x) - \frac{4}{9}\int e^{2x}\cos 3x\, dx$$
$$\frac{13}{9}\int e^{2x}\cos 3x\, dx = \frac{1}{9}e^{2x}(3\sin 3x + 2\cos 3x)$$
$$\int e^{2x}\cos 3x\, dx = \frac{1}{13}e^{2x}(3\sin 3x + 2\cos 3x)$$
$$\int_{-2}^{3} e^{2x}\cos 3x\, dx = \left[\frac{1}{13}e^{2x}(3\sin 3x + 2\cos 3x)\right]_{-2}^{3}$$
$$= \frac{1}{13}[e^6(3\sin 9 + 2\cos 9) - e^{-4}(3\sin(-6) + 2\cos(-6))]$$
$$= \frac{1}{13}[e^6(2\cos 9 + 3\sin 9) - e^{-4}(2\cos 6 - 3\sin 6)]$$
$$\approx -18.186$$

Check: $\text{NINT}\left(e^{2x}\cos 3x, x, -2, 3\right) \approx -18.186$

29. $y = \int x^2 e^{4x}\, dx$

Let $u = x^2$ $\quad dv = e^{4x}dx$

$du = 2x\, dx$ $\quad v = \frac{1}{4}e^{4x}$

$$y = (x^2)\left(\frac{1}{4}e^{4x}\right) - \int\left(\frac{1}{4}e^{4x}\right)(2x\, dx)$$
$$= \frac{1}{4}x^2e^{4x} - \frac{1}{2}\int xe^{4x}dx$$

Let $u = x$ $\quad dv = e^{4x}\, dx$

$du = dx$ $\quad v = \frac{1}{4}e^{4x}$

$$y = \frac{1}{4}x^2e^{4x} - \frac{1}{2}\left[(x)\left(\frac{1}{4}e^{4x}\right) - \int\left(\frac{1}{4}e^{4x}\right)dx\right]$$
$$y = \frac{1}{4}x^2e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} + C$$
$$y = \left(\frac{x^2}{4} - \frac{x}{8} + \frac{1}{32}\right)e^{4x} + C$$

33. Let $u = x$ $\quad dv = \sin x\, dx$

$du = dx$ $\quad v = -\cos x$

$$\int x\sin x\, dx = -x\cos x + \int \cos x\, dx$$
$$= -x\cos x + \sin x + C$$

(a) $\int_0^{\pi}|x\sin x|\, dx = \int_0^{\pi} x\sin x\, dx$
$$= [-x\cos x + \sin x]_0^{\pi}$$
$$= -\pi(-1) + 0 + 0(1) - 0$$
$$= \pi$$

(b) $\int_{\pi}^{2\pi}|x\sin x|\, dx = -\int_{\pi}^{2\pi} x\sin x\, dx$
$$= [x\cos x - \sin x]_{\pi}^{2\pi}$$
$$= 2\pi(1) - 0 - \pi(-1) + 0$$
$$= 3\pi$$

(c) $\int_{\pi}^{2\pi}|x\sin x|\, dx = \int_0^{\pi}|x\sin x|\, dx + \int_{\pi}^{2\pi}|x\sin x|\, dx$
$$= \pi + 3\pi = 4\pi$$

39. B. $\int x\sin(5x)\, dx$

$dv = \sin(5x)\, dx$ $\quad v = \int \sin(5x)\, dx = -\frac{1}{5}\cos 5x$

$u = x$ $\quad du = dx$

$$-\frac{1}{5}x\cos(5x) - \int -\frac{1}{5}\cos(5x)\, dx = -\frac{1}{5}x\cos x + \frac{1}{25}\sin(5x)$$

41. C. $\int dy = \int 4x\ln x\, dx$

$dv = 4x\, dx$ $\quad v = \int 4x\, dx = 2x^2$

$u = \ln x$ $\quad du = \frac{1}{x}\, dx$

$$2x^2\ln x - \int 2x^2\frac{1}{x}\, dx = 2x^2\ln x - x^2 + C$$