6.4 day 2

Section 6.4 Exercises

1. $\int y\,dy = \int x\,dx$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$
$$(2)^2 = (1)^2 + C$$
$$C = 3$$

$y = \sqrt{x^2+3}$, valid for all real numbers

3. $\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$

$$\ln y = \ln x + C$$
$$y = x + C$$
$$2 = 2 + C$$
$$C = 0$$

$y = x$, valid on the interval $(0, \infty)$

5. $\int \frac{dy}{y+5} = \int (x+2)\,dx$

$$\ln (y+5) = \frac{x^2}{2} + 2x + C$$
$$y = e^{x^2/2+2x+C} - 5$$
$$y = e^c e^{x^2/2+2x} - 5 = Ae^{x^2/2+2x} - 5$$

$y = 6\,e^{x^2/2+2x} - 5$, valid for all real numbers

7. $\frac{dy}{dx} = \cos x\; e^y e^{\sin x}$

$$\int e^{-y}\,dy = \int \cos x\; e^{\sin x}\,dx$$
$$-e^{-y} = e^{\sin x} + C$$
$$-e^0 = e^{\sin 0} + C$$
$$C = -2$$

$y = -\ln(2 - e^{\sin x})$, valid for all real numbers.

11. $y(t) = y_0 e^{kt}$

$y(t) = 100e^{1.5t}$

13.

$$y(t) = y_0 e^{kt}$$
$$y(t) = 50e^{kt}$$
$$y(5) = 100 = 50e^{5k}$$
$$2 = e^{5k}$$
$$\ln 2 = 5k$$
$$k = 0.2 \ln 2$$

Solution : $y(t) = 50e^{(0.2 \ln 2)t}$ or $y(t) = 50 \cdot 2^{0.2t}$

15. Doubling time:

$$A(t) = A_0 e^{rt}$$
$$2000 = 1000e^{0.086t}$$
$$2 = e^{0.086t}$$
$$\ln 2 = 0.086t$$
$$t = \frac{\ln 2}{0.086} \approx 8.06 \text{ yr}$$

Amount in 30 years:

$$A = 1000e^{(0.086)(30)} \approx \$13{,}197.14$$

17. Initial deposit:

$$A(t) = A_0 e^{rt}$$
$$2898.44 = A_0 e^{(0.0525)(30)}$$
$$A_0 = \frac{2898.44}{e^{1.575}} \approx \$600.00$$

Doubling time:

$$A(t) = A_0 e^{rt}$$
$$1200 = 600e^{0.0525t}$$
$$2 = e^{0.0525t}$$
$$\ln 2 = 0.0525t$$
$$t = \frac{\ln 2}{0.0525} \approx 13.2 \text{ years}$$

21. $\frac{dy}{dt} = -0.0077y$

$$\int \frac{1}{y}\,dy = \int -0.0077\,dt$$
$$\ln y = -0.0077t$$
$$t = \frac{\ln (1/2)}{-0.0077} = 90 \text{ years}$$

23. **(a)** Since there are 48 half-hour doubling times in 24 hours, there will be $2^{48} \approx 2.8 \times 10^{14}$ bacteria.

(b) The bacteria reproduce fast enough that even if many are destroyed there are still enough left to make the person sick.

25. $0.9 = e^{-0.18t}$

$$\ln 0.9 = -0.18t$$
$$t = -\frac{\ln 0.9}{0.18} \approx 0.585 \text{ days}$$

6.4 day 2 continued...

31. (a) First, we find the value of k.

$$T - T_s = (T_0 - T_s)e^{-kt}$$
$$60 - 20 = (90 - 20)e^{-(k)(10)}$$
$$\frac{4}{7} = e^{-10k}$$
$$k = -\frac{1}{10}\ln\frac{4}{7}$$

When the soup cools to 35°, we have:

$$35 - 20 = (90 - 20)e^{[(1/10)\ \ln\ (4/7)]t}$$
$$15 = 70e^{[(1/10)\ \ln\ (4/7)]t}$$
$$\ln\frac{3}{14} = \left(\frac{1}{10}\ln\frac{4}{7}\right)t$$
$$t = \frac{10\ln\left(\frac{3}{14}\right)}{\ln\left(\frac{4}{7}\right)} \approx 27.53 \text{ min}$$

It takes a total of about 27.53 minutes, which is an additional 17.53 minutes after the first 10 minutes.

(b) Using the same value of k as in part (a), we have:

$$T - T_s = (T_0 - T_s)e^{-kt}$$
$$35 - (-15) = [90 - (-15)]e^{[(1/10)\ \ln\ (4/7)]t}$$
$$50 = 105e^{[(1/10)\ \ln\ (4/7)]t}$$
$$\ln\frac{10}{21} = \left(\frac{1}{10}\ln\frac{4}{7}\right)t$$
$$t = \frac{10\ln\left(\frac{10}{21}\right)}{\ln\left(\frac{4}{7}\right)} \approx 13.26$$

It takes about 13.26 minutes

35. Use $k = \frac{\ln 2}{5700}$ (see Example 3).

$$e^{-kt} = 0.445$$
$$-kt = \ln 0.445$$
$$t = -\frac{\ln 0.445}{k} = -\frac{5700\ln 0.445}{\ln 2} \approx 6658 \text{ years}$$

Crater Lake is about 6658 years old.

41. (a) $\frac{dp}{dh} = kp$

$$\frac{dp}{p} = k\ dh$$
$$\int\frac{dp}{p} = \int k\ dh$$
$$\ln|p| = kh + C$$
$$e^{\ln|p|} = e^{kh+C}$$
$$|p| = e^C e^{kh}$$
$$p = Ae^{kh}$$

Initial condition: $p = p_0$ when $h = 0$

$$p_0 = Ae^0$$
$$A = p_0$$

Solution: $p = p_0 e^{kh}$

Using the given altitude-pressure data, we have $p_0 = 1013$ millibars, so:

$$p = 1013e^{kh}$$
$$90 = 1013e^{(k)(20)}$$
$$\frac{90}{1013} = e^{20k}$$
$$k = \frac{1}{20}\ln\frac{90}{1013} \approx -0.121\text{km}^{-1},$$

Thus, we have $p \approx 1013e^{-0.121h}$

(b) At 50 km, the pressure is $1013e^{((1/20)\ln(90/1013))(50)} \approx 2.383$ millibars.

(c) $900 = 1013e^{kh}$

$$\frac{900}{1013} = e^{kh}$$
$$h = \frac{1}{k}\ln\frac{900}{1013} = \frac{20\ln(900/1013)}{\ln(90/1013)} \approx 0.977 \text{ km}$$

The pressure is 900 millibars at an altitude of about 0.977 km.