7.1 Integral as Net Change

Linear Motion Revisited

Position $= S(t) = x(t) = \int v(t) \cdot dt$

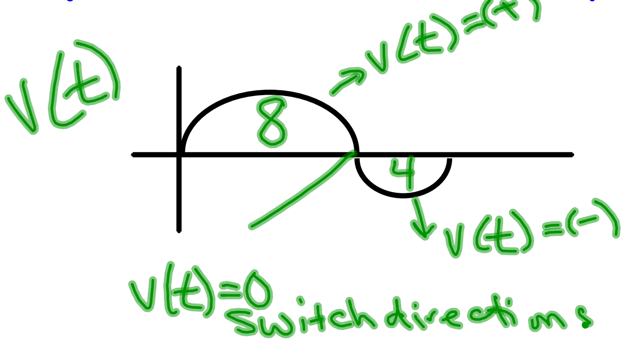
Velocity $= S'(t) = v(t) = \int a(t) \cdot dt$

Acceleration $= S''(t) = V'(t) = a(t)$

- How far are you from your starting point?

$\int_a^b r(t)$

Displacement = Final position - initial position



$s(0)=0$

$8-4=\boxed{4}$

Total Distance Traveled = $8+4=\boxed{12}$

$$\int_a^b |v(t)|dt =$$

Ex 1) $v(t) = 6t^2 - 18t + 12,\ 0 \le t \le 2$

$v(t) = +$ $v(t) = -$ $v(t) = 0$

a.) Determine when the particle is moving to the right, left, stopped.

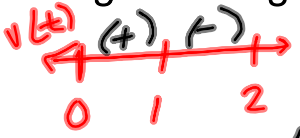
$6t^2 - 18t + 12 = 0$

$6(t^2 - 3t + 2) = 0$

$6(t-1)(t-2) = 0$

$t = 1$

$t = 2$



$[0,1)$ $(1,2)$ $t=1$ $t=2$

b.) Find the particles displacement.

final position - initial position

$$\int_0^2 (6t^2 - 18t + 12)\,dt = 2t^3 - 9t^2 + 12t \Big|_0^2 = (16 - 36 + 24) - (0)$$

$$= \boxed{4}$$

c.) If s(0)=3, find its final position.

$$3 + 4 = \boxed{7}$$

d.) Find the total distance traveled.

$$\int_0^2 (|6t^2 - 18t + 12|)\,dt = \int_0^1 (6t^2 - 18t + 12)\,dt + \left| \int_1^2 (6t^2 - 18t + 12)\,dt \right|$$

$$2t^3 - 9t^2 + 12t \Big|_0^1 + \left| 2t^3 - 9t^2 + 12t \Big|_1^2 \right|$$

$$(2 - 9 + 12) - 0 + \left| (16 - 36 + 24) - (2 - 9 + 12) \right|$$

$$5 + |4 - 5|$$

$$5 + 1 = \boxed{6}$$

e.) Find the acceleration at t=0.

$v(t) = 6t^2 - 18t + 12$

$a(t) = 12t - 18$

$a(0) = 12(0) - 18$

$= \boxed{-18}$

Ex 2) $V = 6\sin(3t), \quad 0 \le t \le (\pi/2)$

$v=+ \quad v=- \quad v=0$

a.) Determine when the particle is moving to the right, left, stopped.

$6\sin(3t)=0$

$\sin(3t)=0$

$3t=0 \qquad 3t=\pi \qquad \cancel{3t=2\pi}$

$t=0 \qquad t=\frac{\pi}{3} \qquad \cancel{t=\frac{2\pi}{3}}$

$(0,\pi/3) \quad (\frac{\pi}{3},\pi/2)$

$+ \quad -$

$0 \quad \frac{\pi}{3} \quad \frac{\pi}{2}$

$t=0$

$t=\frac{\pi}{3}$

b.) Find the particles displacement.

final position − inital position

$6\int_0^{\pi/2}(\sin(3t)dt) = 6\cdot\frac{1}{3}\int \sin u\,du = -2\cos u$

$u=3t$

$du=3dt$

$\frac{1}{3}du=dt$

$= -2\cos(3t)\Big|_0^{\pi/2}$

$= -2\cdot 0 - -2\cdot 1$

$= 0+2$

$= \boxed{2}$

c.) If s(0)=3, find its final position.

$3+2=\boxed{5}$

d.) Find the total distance traveled.

$\int_0^{\pi/2}|v(t)|dt = \int_0^{\pi/3} + \left|\int_{\pi/3}^{\pi/2}\right| = -2\cos(3t)\Big|_0^{\pi/3} + \left|-2\cos(3t)\Big|_{\pi/3}^{\pi/2}\right|$

$= (2 - -2) + |0-2|$

$= 4 + 2 = \boxed{6}$

e.) Find the acceleration at t=0.

$v(t) = 6\sin(3t)$

$v'(t) = a(t) = 6\cos(3t)\cdot 3$

$a(0) = 18\cos(3t)$

$= 18\cos(0)$

$= 18\cdot 1$

$= \boxed{18}$

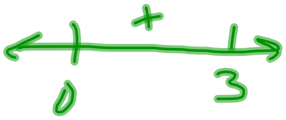
Ex 3) $v = \frac{t}{1+t^2}$ $0 \leq t \leq 3$

(0,3) - never

- $t=0$

a.) Determine when the particle is moving to the right, left, stopped.

$\frac{t}{1+t^2} = 0$ $t=0$



b.) Find the particles displacement.

$$\int_0^3 \frac{t}{1+t^2}\,dt = \begin{array}{l} u = 1+t^2 \\ du = 2t \cdot dt \\ \frac{1}{2}du = t\,dt \end{array} \Bigg| \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln(1+t^2)\Big|_0^3$$

$$= \frac{1}{2}\ln 10 - \cancel{\frac{1}{2}\ln 1}_{\,0}$$

$$= \boxed{\frac{1}{2}\ln 10}$$

c.) If s(0)=3, find its final position.

$$\boxed{3 + \frac{1}{2}\ln 10}$$

d.) Find the total distance traveled.

$$\frac{1}{2}\ln 10$$

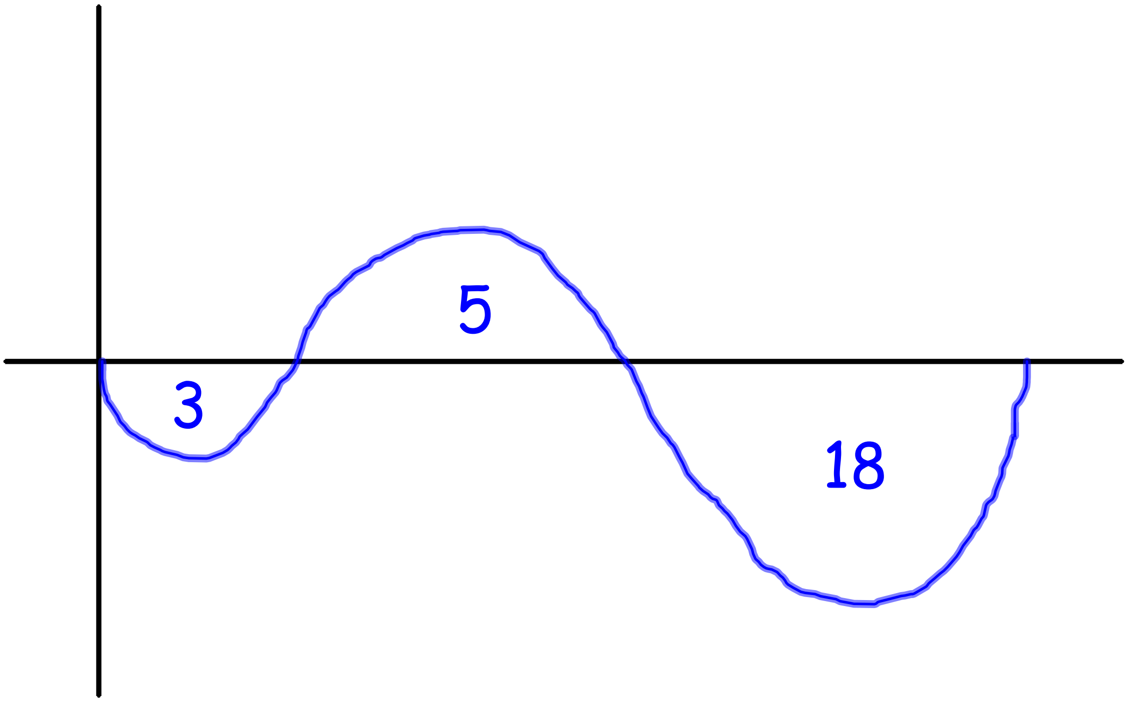
e.) Find the acceleration at t=0.

$$v(t) = \frac{t}{1+t^2}$$

$$a(t) = \frac{(1+t^2)(1) - 2t \cdot t}{(1+t^2)^2}$$

$$a(0) = \frac{(1+0^2)(1) - 2(0)^2}{(1+0^2)^2} = \frac{1-0}{1}$$

$$= \boxed{1}$$


3
5
18

Homework

7.1 #1, 3, 5, 7, 9