Section 7.1 Exercises

1. (a) Right when $v(t) > 0$, which is when $\cos t > 0$, i.e., when $0 \le t < \frac{\pi}{2}$ or $\frac{3\pi}{2} < t \le 2\pi$. Left when $\cos t < 0$, i.e., when $\frac{\pi}{2} < t < \frac{3\pi}{2}$. Stopped when $\cos t = 0$, i.e., when $t = \frac{\pi}{2}$ or $\frac{3\pi}{2}$.

(b) Displacement =

$$\int_0^{2\pi} 5\cos t\,dt = 5\left[\sin t\right]_0^{2\pi} = 5\left[\sin 2\pi - \sin 0\right] = 0$$

(c) Distance $= \int_0^{2\pi} |5\cos t|\,dt$

$$= \int_0^{\pi/2} 5\cos t\,dt + \int_{\pi/2}^{3\pi/2} -5\cos t\,dt + \int_{3\pi/2}^{2\pi} 5\cos t\,dt$$

$= 5 + 10 + 5 = 20$

3. (a) Right when $v(t) = 49 - 9.8t > 0$, i.e., when $0 \le t < 5$. Left when $49 - 9.8t < 0$, i.e., when $5 < t \le 10$. Stopped when $49 - 9.8t = 0$, i.e., when $t = 5$.

(b) Displacement $= \int_0^{10} (49 - 9.8t)\,dt$

$$= \left[49t - 4.9t^2\right]_0^{10} = 49\left[(10 - 10) - 0\right] = 0$$

(c) Distance $= \int_0^{10} |49 - 9.8t|\,dt$

$$= \int_0^5 (49 - 9.8t)\,dt + \int_5^{10} (-49 + 9.8t)\,dt$$

$= 122.5 + 122.5 = 245$

5. (a) Right when $v(t) > 0$, which is when $\sin t \ne 0$ and $\cos t > 0$, i.e., when $0 < t < \frac{\pi}{2}$ or $\frac{3\pi}{2} < t < 2\pi$. Left when $\sin t \ne 0$ and $\cos t < 0$, i.e., when $\frac{\pi}{2} < t < \pi$ or $\pi < t < \frac{3\pi}{2}$. Stopped when $\sin t = 0$ or $\cos t = 0$, i.e., when $t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, or 2π.

(b) Displacement $= \int_0^{2\pi} 5\sin^2 t\cos t\,dt = 5\left[\frac{1}{3}\sin^3 t\right]_0^{2\pi}$

$= 5[0 - 0] = 0$

(c) Distance $= \int_0^{2\pi} |5\sin^2 t\cos t|\,dt$

$$= \int_0^{\pi/2} 5\sin^2 t\cos t\,dt + \int_{\pi/2}^{3\pi/2} -5\sin^2 t\cos t\,dt + \int_{3\pi/2}^{2\pi} 5\sin^2 t\cos t\,dt$$

$$= \frac{5}{3} + \frac{10}{3} + \frac{5}{3} = \frac{20}{3}$$

7. (a) Right when $v(t) > 0$, which is when $\cos t > 0$, i.e., when $0 \le t < \frac{\pi}{2}$ or $\frac{3\pi}{2} < t \le 2\pi$. Left when $\cos t < 0$, i.e., when $\frac{\pi}{2} < t < \frac{3\pi}{2}$. Stopped when $\cos t = 0$, i.e., when $t = \frac{\pi}{2}$ or $\frac{3\pi}{2}$.

(b) Displacement $= \int_0^{2\pi} e^{\sin t}\cos t\,dt = \left[e^{\sin t}\right]_0^{2\pi}$

$= [e^0 - e^0] = 0$

(c) Distance $= \int_0^{2\pi} |e^{\sin t}\cos t|\,dt = \int_0^{\pi/2} e^{\sin t}\cos t\,dt +$

$$\int_{\pi/2}^{3\pi/2} -e^{\sin t}\cos t\,dt + \int_{3\pi/2}^{2\pi} e^{\sin t}\cos t\,dt$$

$$= (e - 1) + \left(e - \frac{1}{e}\right) + \left(1 - \frac{1}{e}\right) = 2e - \frac{2}{e} \approx 4.7$$

9. (a) $v(t) = \int a(t)\,dt = t + 2t^{3/2} + C$, and since $v(0) = 0$, $v(t) = t = +2t^{3/2}$. Then $v(9) = 9 + 2(27) = 63$ mph.

(b) First convert units:

$t + 2t^{3/2}$ mph $= \frac{t}{3600} + \frac{t^{3/2}}{1800}$ mi / sec. Then

Distance $= \int_0^9 \left(\frac{t}{3600} + \frac{t^{3/2}}{1800}\right) dt$

$$= \left[\frac{t^2}{7200} + \frac{t^{5/2}}{4500}\right]_0^9 = \left[\left(\frac{9}{800} + \frac{27}{500}\right) - 0\right] = 0.06525\text{ mi}$$

$= 344.52$ ft.

11. (a) $v(t) = \int a(t)\,dt = \int -32\,dt = -32t + C_1$, where $C_1 = v(0) = 90$.
Then $v(3) = -32(3) + 90 = -6$ ft / sec.

(b) $s(t) = \int v(t)\,dt = -16t^2 + 90t + C_2$, where $= s(0) = 0$. Solve $s(t) = 0$:

$-16t^2 + 90t = 2t(-8t + 45) = 0$

when $t = 0$ or $t = \frac{45}{8} = 5.625$ sec.

The projectile hits the ground at 5.625 sec.

(c) Since starting height = ending height, Displacement = 0.

(d) Max. Height $= s\left(\frac{5.625}{2}\right)$

$$= -16\left(\frac{5.625}{2}\right)^2 + 90\left(\frac{5.625}{2}\right) = 126.5625,$$ and

Distance = 2(Max. Height) = 253.125 ft.

12. Displacement $= \int_0^c v(t)\,dt = -4+5-24 = -23\text{cm}$

13. Total distance $= \int_0^c |v(t)|\,dt = 4+5+24 = 33\text{cm}$

14. At $t=a$, $s = s(0) + \int_0^a v(t)\,dt = 15-4 = 11$.

At $t=b$, $s = s(0) + \int_0^b v(t)\,dt = 15-4+5 = 16$.

At $t=c$, $s = s(0) + \int_0^c v(t)\,dt = 15-4+5-24 = -8$.

15. At $t=a$, where $\frac{dv}{dt}$ is at a maximum (the graph is steepest upward).

16. At $t=c$, where $\frac{dv}{dt}$ is at a maximum (the graph is steepest upward).

17. Distance = Area under curve $= 4\left(\frac{1}{2}\cdot 1\cdot 2\right) = 4$

(a) Final position = Initial position + Distance $= 2+4 = 6$; ends at $x = 6$.

(b) 4 meters

19. (a) Final position $= 2 + \int_0^7 v(t)\,dt$

$$= 2 - \frac{1}{2}(1)(2) + \frac{1}{2}(1)(2) + 1(2) + \frac{1}{2}(2)(2) - \frac{1}{2}(2)(1) = 5;$$

end at $x = 5$.

(b) $\int_0^7 |v(t)|\,dt = \frac{1}{2}(1)(2) + \frac{1}{2}(1)(2) + 1(2) + \frac{1}{2}(2)(2) + \frac{1}{2}(2)(1) = 7$ meters

21.

$$\int_0^{10} 270.8\cdot e^{t/25}\,dt = 27.08\left[25e^{t/25}\right]_0^{10} = 27.08\left[25e^{0.4} - 25\right]$$

≈ 332.965 billion barrels

23. (a) solve $10{,}000\,(2-r) = 0$: $r = 2$ miles.

(b) Width $= \Delta r$: Length $= \pi r$: Area $= 2\pi r\Delta r$

(c) Population = Population density × Area

(d) $\int_0^2 10{,}000(2-r)(2\pi r)\,dr = 20{,}000\pi\int_0^2 \left(2r - r^2\right)dr$

$$= 20{,}000\pi\left[r^2 - \frac{1}{3}r^3\right]_0^2 = 20{,}000\pi\left[\left(4-\frac{8}{3}\right) - 0\right]$$

$$= \frac{80{,}000}{3}\pi \approx 83{,}776$$

31. False. The displacement is the integral of the velocity from $t=0$ to $t=5$ and is positive.

33. C. $(12)(50)(6) = 3600$.

34. D. $5 + \frac{15}{10}\left(4 + 2(8) + 2(6) + 2(9) + 2(10) + 10\right) = 125$.