AP Calculus

CHAPTER 5 WORKSHEET

INTEGRALS

ANSWER KEY

Net Change

1. $\int_{5}^{10} w'(t)\,dt$ is the net change in the child's weight measured in pounds from 5 to 10 years of age.

2. $\int_{0}^{120} R(t)\,dt$ is the amount of oil leaked from the tank measured in gallons from 0 to 120 minutes.

3. $100+\int_{0}^{15} n'(t)\,dt$ is the bee population measured in bees 15 weeks later.

4. Units for $\frac{dm}{dx}$ are pounds per foot per foot or pounds per foot^2

   Units for $\int_{2}^{8} m(x)\,dx$ are pounds.

5.

   a) Since $M(6)=5\sqrt{6}\cos\left(\frac{6}{5}\right)\approx 4.438>0$, the number of mosquitoes is increasing at $t=6$.

   b) $\int_{0}^{31} M(t)\,dt=\int_{0}^{31}\left(5\sqrt{t}\cos\left(\frac{t}{5}\right)\right)dt\approx -35.665$. The mosquito population decrease by 36 individuals.

   c) $1000-\int_{0}^{31} M(t)\,dt\approx 964.335$. There are 964 mosquitoes at $t=31$.

---

6. $\int_{0}^{4} D(x)\,dx=\int_{0}^{4}\left(9+2\sqrt{x}\right)dx=\frac{140}{3}=46\frac{2}{3}$ kilograms

---

7.

   a) $\int_{0}^{24} R(t)\,dt\approx 6\times 10.4+6\times 11.2+6\times 11.3+6\times 10.2=258.6$ gallons. This is the total amount of water that flowed from the pipe in the 24-hour period.

   b) The initial amount in the tank added to the amount that flowed from the pipe yield $115+\int_{0}^{24} R(t)\,dt\approx 373.6$ gallons. This is more than the capacity of the tank, so that tank will not be able to store all the water.

---

8. $\int_{2000}^{4000} C'(x)\,dx = \int_{2000}^{4000} \left(3 - 0.01x + 0.000006x^2\right)dx = 58{,}000$ dollars. Increasing production from 2000 yards to 4000 yards means an added cost of \$58,000.

9.

a) $\int_{9}^{17} E(t)\,dt = \int_{9}^{17} \frac{15600}{t^2 - 24t + 160}\,dt \approx 6004.270$. So 6004 people entered the park by 5 PM.

b) $15 \times \int_{9}^{17} E(t)\,dt + 11 \times \int_{17}^{23} E(t)\,dt = 15 \times \int_{9}^{17} \frac{15600}{t^2 - 24t + 160}\,dt + 11 \times \int_{17}^{23} \frac{15600}{t^2 - 24t + 160}\,dt \approx 104{,}048.165$.

Therefore , the amount collected was \$104,048.

OR

$\int_{9}^{17} E(t)\,dt = \int_{9}^{17} \frac{15600}{t^2 - 24t + 160}\,dt \approx 6004.270$, 6004 people entered the park by 5 PM and

$\int_{17}^{23} E(t)\,dt = \int_{17}^{23} \frac{15600}{t^2 - 24t + 160}\,dt \approx 1271.283$, 1271 people entered the park between 5 PM and 11 PM. Therefore, the amount collected was $15 \times 6004 + 11 \times 1271 = \$104{,}041$

c) $E(17) - L(17) = -380.281$ people per hour. Since this rate is negative, the amount of people in the park is decreasing (people are leaving the park faster than entering in at $t = 17$.)

d) $\int_{9}^{17} \left(E(t) - L(t)\right)dt = 3725$ is the net change in the number of people in the park since it opened until 5 PM. Also, since when the park opened there were no people in it, this is also the number of people in the park at 5 PM.

10.

a) $R'(45) \approx \frac{R(50) - R(40)}{50 - 40} = 1.5$ gallons/minute^2

b) $\int_{0}^{90} R(t)\,dt \approx 30 \times 20 + 10 \times 30 + 10 \times 40 + 20 \times 55 + 20 \times 65 = 3700$. This is the total number of gallons of fuel consumed by the airplane during the 90 minutes of flight.

c) The approximation of 3700 gallons is less than the actual value of $\int_{0}^{90} R(t)\,dt$, because $R(t)$ is strictly increasing on $0 \le t \le 90$.

d) $\frac{1}{90 - 0} \int_{0}^{90} R(t)\,dt$ is the average of the rate of fuel consumption measured in gallons/minute during the 90 minutes flight.
