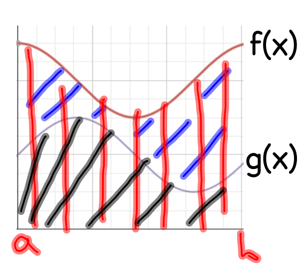
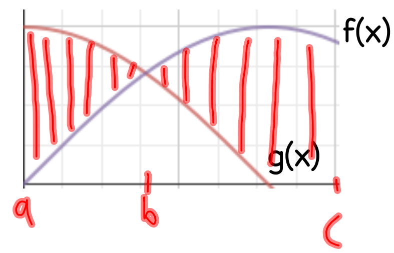
7.2 Areas in the Plane

Find the area of the shaded region.

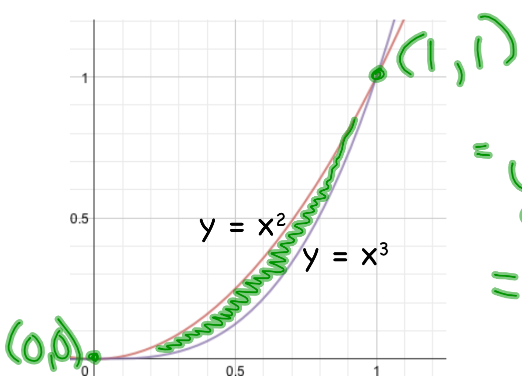


$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b \left(f(x)-g(x)\right)dx$$



$$\int_a^b \left(g(x)-f(x)\right)dx + \int_b^c \left(f(x)-g(x)\right)dx$$

Ex 1) Find the area between the curves.



$$= \int_0^1 (x^2 - x^3)\,dx$$

$$= \frac{x^3}{3} - \frac{x^4}{4}\Big|_0^1 = \left(\frac{1}{3}-\frac{1}{4}\right) - (0-0)$$

$$= \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$$

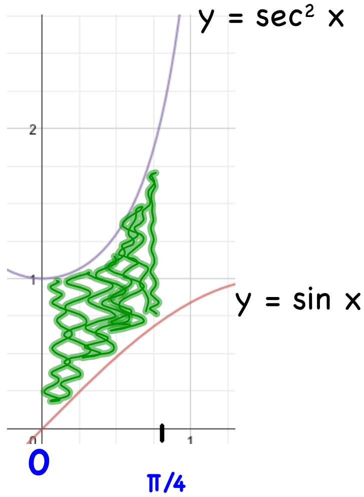
$x^3 = x^2$

$x^3 - x^2 = 0$

$x^2(x-1) = 0$

$x=0 \quad x=1$

Ex 2) Find the area between the curves. [0, π/4]

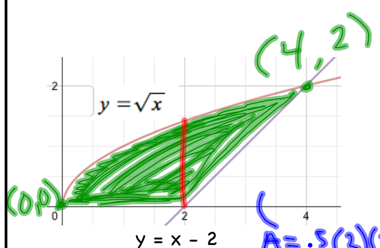


$$\int_0^{\pi/4} (\sec^2 x - \sin x)\,dx$$

$$= \tan x + \cos x \Big|_0^{\pi/4}$$

$$= \left(1 + \frac{\sqrt{2}}{2}\right) - (0+1) = \frac{\sqrt{2}}{2}$$

Ex 3) Find the area between the curves and the x-axis.



$x^{1/2}$

$$\int_0^4 \sqrt{x}\,dx - \int_2^4 (x-2)\,dx$$

$A = .5(2)(2)$
$= 2$

$(\sqrt{x})^2 = (x-2)^2$

$x = x^2 - 4x + 4$

$0 = x^2 - 5x + 4$

$0 = (x-1)(x-4)$

$\underline{x = 4}$

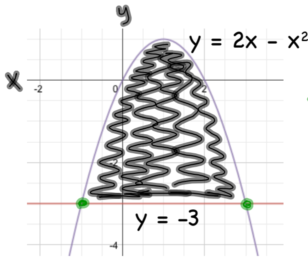
$$\frac{2}{3}x^{3/2}\Big|_0^4 - 2$$

$$= \frac{2 \cdot 4^{3/2}}{3} - 0 - 2$$

$$= \frac{2 \cdot 8}{3} - 2$$

$$= \frac{16}{3} - \frac{6}{3} = \frac{10}{3}$$

Ex 4) Find the area enclosed by these graphs.



$$= \int_{-1}^{3} (2x - x^2 - -3)\,dx$$

$$= \int_{-1}^{3} (2x - x^2 + 3)\,dx$$

$$= x^2 - \frac{x^3}{3} + 3x \Big|_{-1}^{3} =$$

$$= (9 - 9 + 9) - \left(1 + \frac{1}{3} - 3\right)$$

$$= 9 - \left(\frac{-5}{3}\right)$$

$$= \boxed{\frac{32}{3}}$$

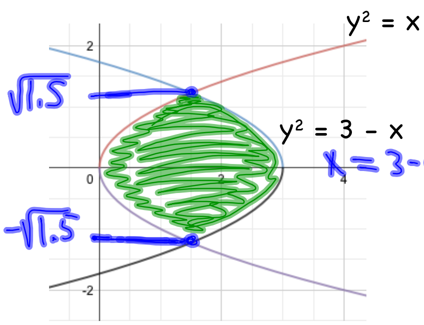
$2x - x^2 = -3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3 \quad x = -1$

Ex 5) Find the area enclosed by these graphs.



$\sqrt{1.5}$

$-\sqrt{1.5}$

$x = 3 - y^2$

$$\int_{-\sqrt{1.5}}^{\sqrt{1.5}} \left(3 - y^2 - (y^2)\right) dy$$

$$\int_{-\sqrt{1.5}}^{+\sqrt{1.5}} (3 - 2y^2)\,dy$$

$y^2 = 3 - y^2$

$2y^2 = 3$

$\sqrt{y^2} = \sqrt{3/2}$

$y = \pm\sqrt{1.5}$

$\text{fnInt}(3 - 2x^2, x, -\sqrt{1.5}, \sqrt{1.5})$

$= 4.899$