Section 7.2 Exercises

1. $\int_0^{\pi}(1-\cos^2 x)\,dx=\left[\frac{1}{2}x-\frac{1}{4}\sin 2x\right]_0^{\pi}=\frac{\pi}{2}$

3. $\int_0^1(y^2-y^3)\,dy=\left[\frac{1}{3}y^3-\frac{1}{4}y^4\right]_0^1=\frac{1}{12}$

5. Use the region's symmetry:

$$2\int_0^2[2x^2-(x^4-2x^2)]dx=2\int_0^2(-x^4+4x^2)dx$$
$$=2\left[-\frac{1}{5}x^5+\frac{4}{3}x^3\right]_0^2$$
$$=2\left[\left(-\frac{32}{5}+\frac{32}{3}\right)-0\right]=\frac{128}{15}$$

9. $\int_0^1\left(x-\frac{x^2}{4}\right)dx+\int_1^2\left(1-\frac{x^2}{4}\right)dx$

$$=\left(\frac{x^2}{2}-\frac{x^3}{12}\right)\Bigg|_0^1+\left(x-\frac{x^3}{12}\right)\Bigg|_1^2$$
$$=\left(\frac{1^2}{2}-\frac{1^3}{12}\right)-\left(\frac{0^2}{12}-\frac{0^3}{12}\right)+\left(2-\frac{2^3}{12}\right)-\left(1-\frac{1^3}{12}\right)$$
$$=5/6$$

11. $\int_{-\sqrt{2}}^{\sqrt{2}}(3-y^2-(y^2-1))\,dy$

$$=\left(3y-\frac{y^3}{3}-\frac{y^3}{3}+y\right)\Bigg|_{-\sqrt{2}}^{\sqrt{2}}$$
$$=4\sqrt{2}-\frac{2(\sqrt{2})^3}{3}-\left(4(-\sqrt{2})-\frac{2(-\sqrt{2})^3}{3}\right)$$
$$=7.542$$

13. $\int_{-2}^{0}(2x^3-x^2-5x)(-x^2+3x)\,dx$

$$+\int_0^2(-x^2+3x-(2x^3-x^2-5x))\,dx$$
$$=\left(\frac{x^4}{2}-\frac{x^3}{3}-\frac{5x^2}{2}+\frac{x^3}{3}+\frac{3x^2}{2}\right)\Bigg|_{-2}^{0}$$
$$+\left(-\frac{x^3}{3}+\frac{3x^2}{2}-\frac{x^4}{2}+\frac{x^3}{3}+\frac{5x^2}{2}\right)\Bigg|_0^2$$
$$=\left(0-\left(\frac{-2^4}{2}-\frac{-2^3}{3}-\frac{5(-2)^2}{2}+\frac{-2^3}{3}+\frac{3(-2)^2}{2}\right)\right)$$
$$+\left(-\frac{2^3}{3}+\frac{3(2)^2}{2}-\frac{2^4}{2}+\frac{2^3}{3}+\frac{5(2)}{2}-0\right)$$
$$=16$$

17. Solve $7-2x^2=x^2+4$: $x^2=1$, so the curves intersect at $x=\pm1$.

$$\int_{-1}^{1}\left[(7-2x^2)-(x^2+4)\right]dx=\int_{-1}^{1}(-3x^2+3)\,dx$$
$$=3\int_{-1}^{1}(1-x^2)\,dx$$
$$=3\left[x-\frac{1}{3}x^3\right]_{-1}^{1}$$
$$=3\left[\frac{2}{3}-\left(-\frac{2}{3}\right)\right]=4$$

23. Solve for x: $x=\frac{y^2}{4}-1$ and $x=\frac{y}{4}+4$.

Now solve $\frac{y^2}{4}-1=\frac{y}{4}+4$: $\frac{y^2}{4}-\frac{y}{4}-5=0$,

$y^2-y-20=(y-5)(y+4)=0$.

The curves intersect at $y=-4$ and $y=5$.

$$\int_{-4}^{5}\left[\left(\frac{y}{4}+4\right)-\left(\frac{y^2}{4}-1\right)\right]dy$$
$$=\int_{-4}^{5}\left(-\frac{y^2}{4}+\frac{y}{4}+5\right)dy$$
$$=\left[-\frac{y^3}{12}+\frac{y^2}{8}+5y\right]_{-4}^{5}$$
$$=\left(-\frac{125}{12}+\frac{25}{8}+25\right)-\left(\frac{16}{3}+2-20\right)=\frac{243}{8}=30\frac{3}{8}$$

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27. Solve for x: $x = 3 - y^2$ and $x = -\frac{y^2}{4}$.

Now solve $= 3 - y^2 = -\frac{y^2}{4}$: $y^2 = 4$,

so the curves intersect at $y = \pm 2$.

Use the region's symmetry:

$$2\int_0^2 \left(3 - y^2 + \frac{y^2}{4}\right) dy = 2\int_0^2 \left(3 - \frac{3y^2}{4}\right) dy$$

$$= 2\left[3y - \frac{y^3}{4}\right]_0^2$$

$$= 2(6-2) - 0 = 8$$

29. Use the region's symmetry:

$$2\int_0^{\pi/3} (8\cos x - \sec^2 x)\,dx = 2\left[8\sin x - \tan x\right]_0^{\pi/3}$$

$$= 2\left[(4\sqrt{3} - \sqrt{3}) - 0\right] = 6\sqrt{3}$$

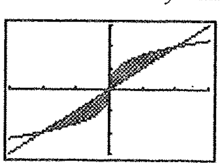
35. $\int_{-3}^{1} \sqrt{x+3}\,dx - \int_0^1 (2x)\,dx$

$$= \frac{2}{3}(x+3)^{3/2}\Big|_{-3}^{1} - x^2\Big|_0^1$$

$$= \frac{2}{3}(1+3)^{3/2} - \frac{2}{3}(-3+3)^{3/2} - (1^2 - 0^2)$$

$$= 4.333$$

37. Solve for x: $x = y^3$ and $x = y$.



[−1.5, 1.5] by [−1.5, 1.5]

The curves intersect at $x = 0$ and $x = \pm 1$. Use the area's

symmetry: $2\int_0^1 (y - y^3)\,dy = 2\left[\frac{1}{2}y^2 - \frac{1}{4}y^4\right]_0^1 = \frac{1}{2}$

53. E. $\int_0^3 (x^2 - (-x))\,dx$.

$$= \left(\frac{x^3}{3} + \frac{x^2}{2}\right)\Big|_0^3 = \frac{27}{2}$$