7.3 day 1

## Volume of a Solid

The volume of a solid can be found by finding the sum of the area of the cross sections.

Area of cross-section

width

$$V = \int_a^b A(x)\,dx.$$

## How to Find Volume by Slicing

1. Sketch the solid and a typical cross section.

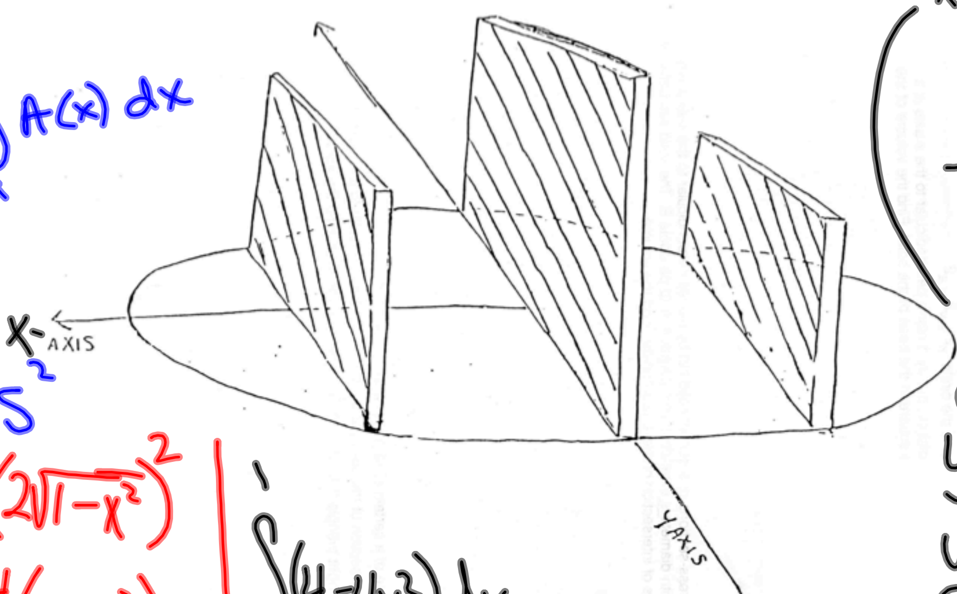
2. Find a formula for the area of the cross section.

3. Find the bounds of integration.

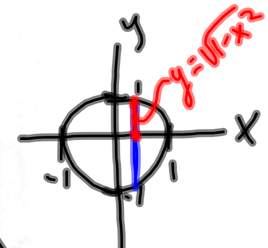
4. Integrate A(x) to find volume.

The circle formed by $x^2 + y^2 = 1$ represents the base of a solid. Squares are stacked perpendicular to the x-axis to form a 3-D object. Find the volume of that solid.

$$V = \int_a^b A(x)\,dx$$




$x^2 + y^2 = 1$




$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$

$y = \sqrt{1-x^2}$

$A = s^2$

$A = \left(2\sqrt{1-x^2}\right)^2$

$A = 4(1-x^2)$

$A = 4 - 4x^2$

Area of the Square Slice

$$\int_{-1}^{1} (4 - 4x^2)\,dx$$

$$= 4x - \frac{4x^3}{3}\Big|_{-1}^{1} = 4(1) - \frac{4(1)^3}{3} - \left(-4 + \frac{4}{3}\right)$$

$$= 8 - \frac{8}{3} = \frac{24}{3} - \frac{8}{3}$$

$$= \frac{16}{3}$$

A solid is made so that its base is the shape of the region between the x-axis and one arch on the curve y = 2sin x. Each cross section cut perpendicular to the x-axis is a semi-circle whose diameter runs from the x-axis to the curve. Find the volume of the solid

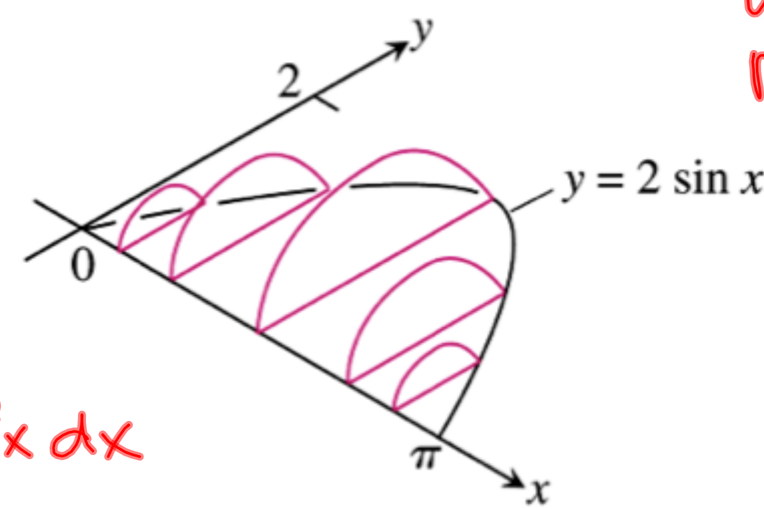
$A = \frac{\pi r^2}{2}$

$A = \frac{\pi (\sin x)^2}{2}$

$d = 2\sin x$

$r = \frac{1}{2} \cdot 2\sin x$

$r = \sin x$




$V = \int_0^{\pi} \frac{\pi}{2} \sin^2 x \, dx$

$\left(\frac{\pi}{2}\right)$fnInt$(\sin(x)^2, x, 0, \pi) \approx \boxed{2.467\ u^3}$

Solids of Revolution

-Formed when a curve or region is revolved around a line.

-The cross section of a solid of revolution is circular.

-These cross sections are either in the shape of a disc or a washer (donut!!!).

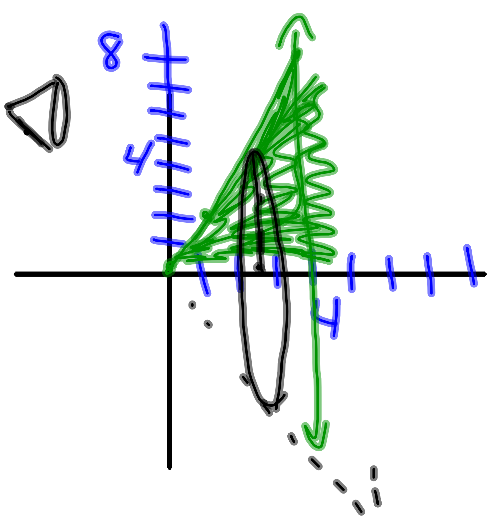
pizza platter

## Discs

Find the volume of the solid generated by revolving the region bounded by the lines and curves about the x-axis.

$A = \pi r^2$

$y=2x \qquad x=4$




$$V = \int_0^4 \pi (2x)^2 \, dx$$

$$= \int_0^4 \pi 4x^2 \, dx = 4\pi \int_0^4 x^2 \, dx$$

$$= 4\pi \cdot \frac{x^3}{3} \Big|_0^4$$

$$= 4\pi \cdot \frac{64}{3} - 0$$

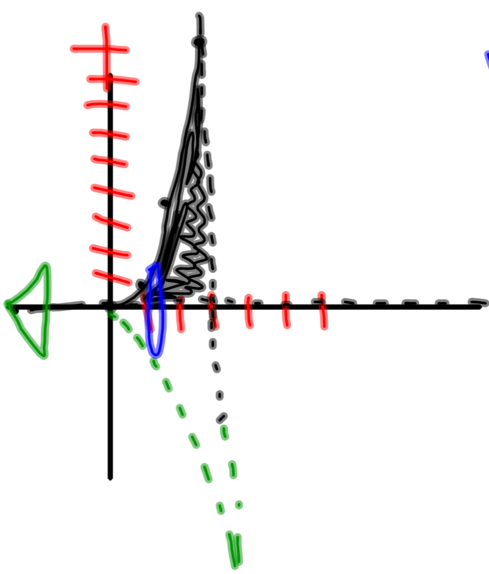
$$= \boxed{\frac{256\pi}{3} \text{ or } 268.08}$$

Find the volume of the solid generated by revolving the region bounded by the lines and curves about the x-axis.

$A = \pi r^2 = \pi (x^2)^2$

$A = \pi x^4$

$y = x^2 \quad y = 0 \quad x = 3$



$V = \int_0^3 \pi x^4 \, dx = \pi \cdot \frac{x^5}{5} \Big|_0^3$

$= \boxed{\frac{243\pi}{5} \approx 152.68 u^3}$

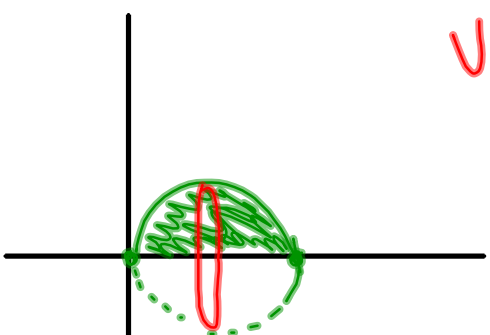
Find the volume of the solid generated by revolving the region bounded by the lines and curves about the x-axis. $(x-x^2)(x-x^2)$

$A = \pi r^2 = \pi(x-x^2)^2$

$y = x - x^2 \qquad y = 0$

$$V = \pi \int_0^1 (x-x^2)^2\,dx = \pi \int_0^1 (x^2 - 2x^3 + x^4)\,dx$$

$$= .1047\,u^3$$



$0 = x - x^2$

$0 = x(1-x)$

$x=0 \quad x=1$

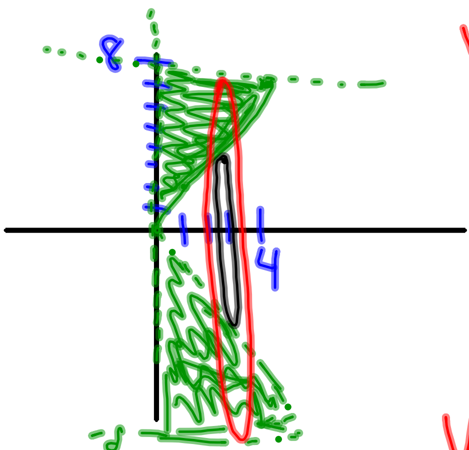
## Washer (Donut Method)

Outer radius − inner radius

$\pi(r_1)^2 - \pi(r_2)^2$

Find the volume of the solid generated by revolving the region bounded by the lines and curves about the x-axis.

$y=2x$ $y=8$ $x=0$




$$V = \int_0^4 \pi(8)^2 - \pi(2x)^2)\,dx$$

$$V = \int_0^4 (64\pi - 4\pi x^2\,dx)$$

$$V = \pi \int_0^4 (64 - 4x^2)\,dx$$

$$V = \pi \left[ \left(64x - \frac{4x^3}{3}\right) \Big|_0^4 \right]$$

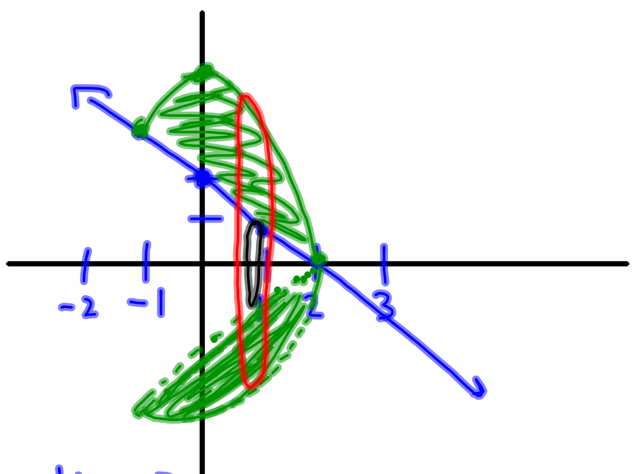
$$V = \pi \left[ 256 - \frac{256}{3} - 0 \right]$$

$$= \pi \left[ \frac{256 \cdot 3}{3} - \frac{256}{3} \right]$$

$$\boxed{= \frac{512\pi}{3} = 536.17\ u^3}$$

Find the volume of the solid generated by revolving the region bounded by the lines and curves about the x-axis.

$y=4-x^2$ $\quad$ $y=2-x$




$4-x^2=2-x$

$0=x^2-x-2$

$0=(x-2)(x+1)$

$x=2 \quad x=^-1$

outer radius $\quad$ inner radius

$$V=\int_{-1}^{2}\pi(4-x^2)^2-\pi(2-x)^2\,dx$$

$$=\pi\int_{-1}^{2}\left(16-8x^2+x^4-(4-4x+x^2)\right)dx$$

$$=\pi\int_{-1}^{2}\left(\left(12-9x^2+x^4+4x\right)\right)dx$$

$$=\pi\left(12x-3x^3+\frac{x^5}{5}+2x^2\right)\Big|_{-1}^{2}$$

$$=\boxed{67.858u^3}$$