Volume of a Solid

The volume of a solid can be found by finding the sum of the area of the cross sections.

$$V = \int_a^b A(x)\,dx.$$

How to Find Volume by Slicing

1. Sketch the solid and a typical cross section.

2. Find a formula for the area of the cross section.

3. Find the bounds of integration.

4. Integrate A(x) to find volume.

Solids of Revolution

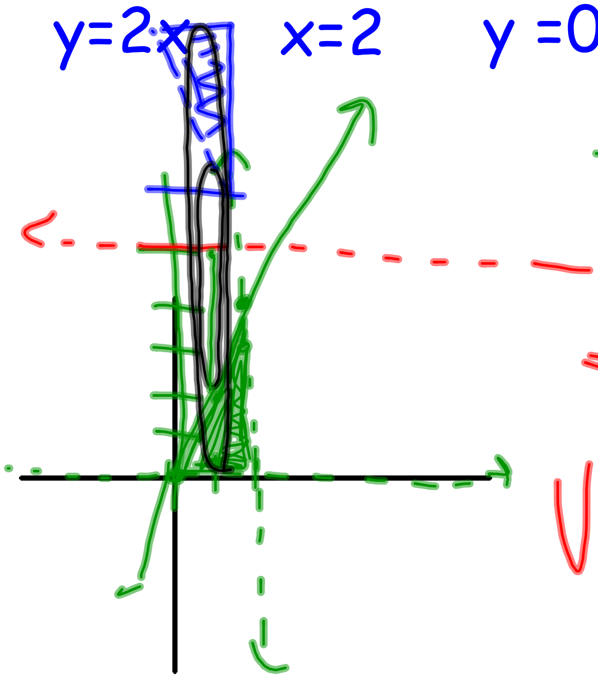
-Formed when a curve or region is revolved around a line.

-The cross section of a solid of revolution is circular.

-These cross sections are either in the shape of a disc or a washer (donut!!!).

Find the volume of the solid generated by revolving the region bounded by the lines and curves about the line y = 5.

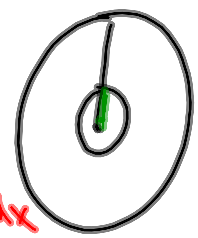
$y=2x$ $\quad x=2$ $\quad y=0$



$(5-2x)(5-2x)$

$$= \int_0^2 \pi(5)^2 - \pi(5-2x)^2 \, dx$$

$$= \pi \int_0^2 (\cancel{25} \cancel{-25} + 20x - 4x^2)\, dx$$



$$V = \pi\left(10x^2 - \frac{4}{3}x^3\right)\Big|_0^2$$

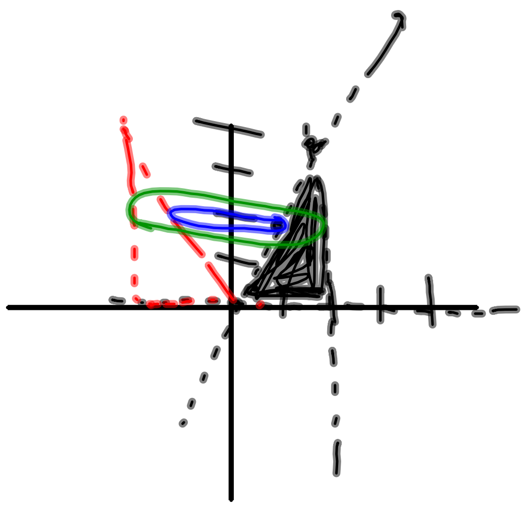
$$= \pi\left(40 - \frac{32}{3} - (0)\right)$$

$$= \pi\left(\frac{120}{3} - \frac{32}{3}\right) = \frac{88\pi}{3} u^3$$

Find the volume of the solid generated by revolving the region bounded by the lines and curves about the line y-axis.

$y=2x \quad x=2 \quad y=0$

$\frac{y}{2}=x$



$$V=\pi\int\left((2)^2-\left(\frac{y}{2}\right)^2\right)dy$$

$$=\pi\int_0^4\left(4-\frac{y^2}{4}\right)dy$$

$$=\pi\left(4y-\frac{y^3}{12}\right)\Big|_0^4$$

$$=\pi\left(16-\frac{64}{12}-(0)\right)$$

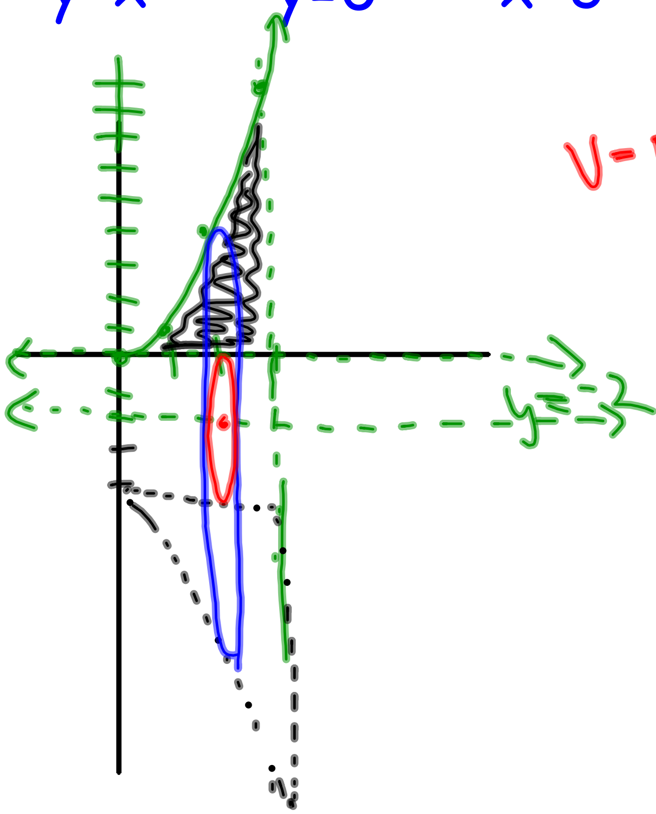
$$=\pi\left(\frac{192}{12}-\frac{64}{12}\right)=\frac{128}{12}\pi=\frac{32}{3}\pi$$

4^3

Find the volume of the solid generated by revolving the region bounded by the lines and curves about the line y = -2.

$A=\pi r^2$

$y=x^2$ $y=0$ $x=3$



$$V=\pi\int_0^3\left((x^2+2)^2-(2)^2\right)dx$$

outer inner

$$V=\pi\int_0^3(x^4+4x^2+4-4)\,dx$$

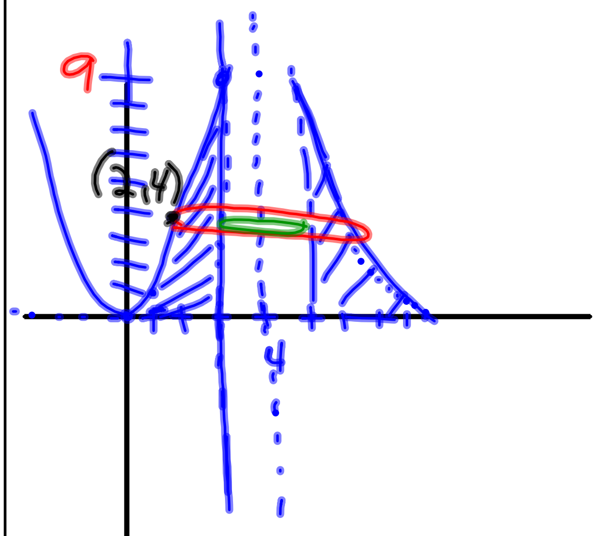
$$=\pi\int_0^3(x^4+4x^2)\,dx$$

$$=\boxed{265.78\,u^3}$$

Find the volume of the solid generated by revolving the region bounded by the lines and curves about the line x = 4.

$x=\sqrt{y}$

$y=x^2$ $y=0$ $x=3$



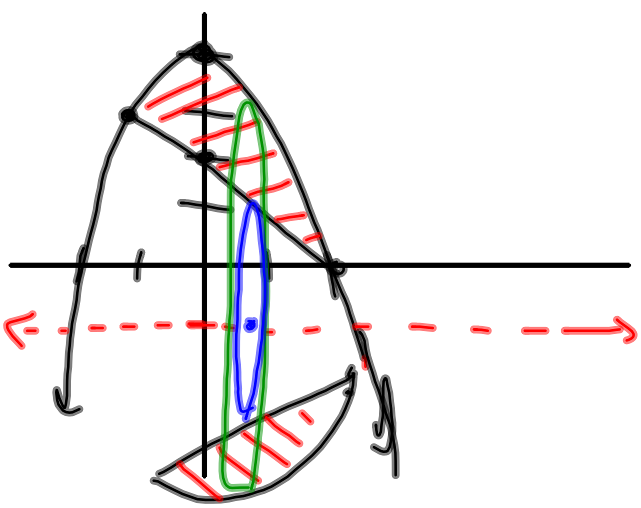
$$V=\pi\int_0^9\left((4-\sqrt{y})^2-(1)^2\,dy\right)$$

$$=\pi\int_0^9\left((4-\sqrt{y})^2-1\right)dy$$

$$\approx 98.96\,u^3$$

Find the volume of the solid generated by revolving the region bounded by the lines and curves about the line y = -1.

$y=4-x^2$ $y=2-x$



$4-x^2=2-x$

$0=x^2-x-2$

$0=(x-2)(x+1)$

$x=2$ $x=-1$

$$V=\pi\int_{-1}^{2}(4-x^2+1)^2-(2-x+1)^2\,dx$$

$$V=\pi\int_{-1}^{2}\left((5-x^2)^2-(3-x)^2\right)dx$$

$$=\boxed{96.13\ u^3}$$