

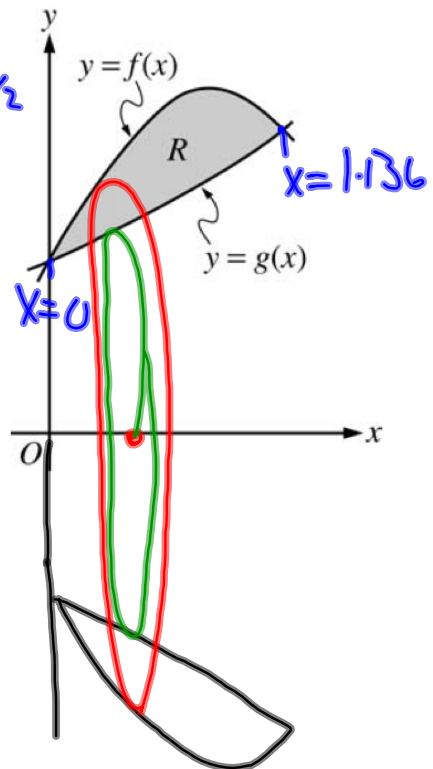
Let  $f$  and  $g$  be the functions given by  $f(x) = 1 + \sin(2x)$  and  $g(x) = e^{x/2}$ . Let  $R$  be the shaded region in the first quadrant enclosed by the graphs of  $f$  and  $g$  as shown in the figure above.

- (a) Find the area of  $R$ .
- (b) Find the volume of the solid generated when  $R$  is revolved about the  $x$ -axis.
- (c) The region  $R$  is the base of a solid. For this solid, the cross sections perpendicular to the  $x$ -axis are semicircles with diameters extending from  $y = f(x)$  to  $y = g(x)$ . Find the volume of this solid.

A)  $A = \int_0^{1.136} (1 + \sin(2x) - e^{x/2}) dx$   
 $= \boxed{.429 \text{ u}^2}$

$1 + \sin(2x) = e^{x/2}$

B)  $V = \pi \int_0^{1.136} (1 + \sin(2x))^2 - (e^{x/2})^2 dx$   
 $= \boxed{4.267 \text{ u}^3}$



C)  $A = \frac{1}{2} \pi r^2$   
 $A = \frac{1}{2} \pi ( \quad )^2$

$V = \frac{1}{2} \pi \int_0^{1.136} \left( \frac{1 + \sin(2x) - e^{x/2}}{2} \right)^2 dx$

$r = \frac{1 + \sin(2x) - e^{x/2}}{2}$

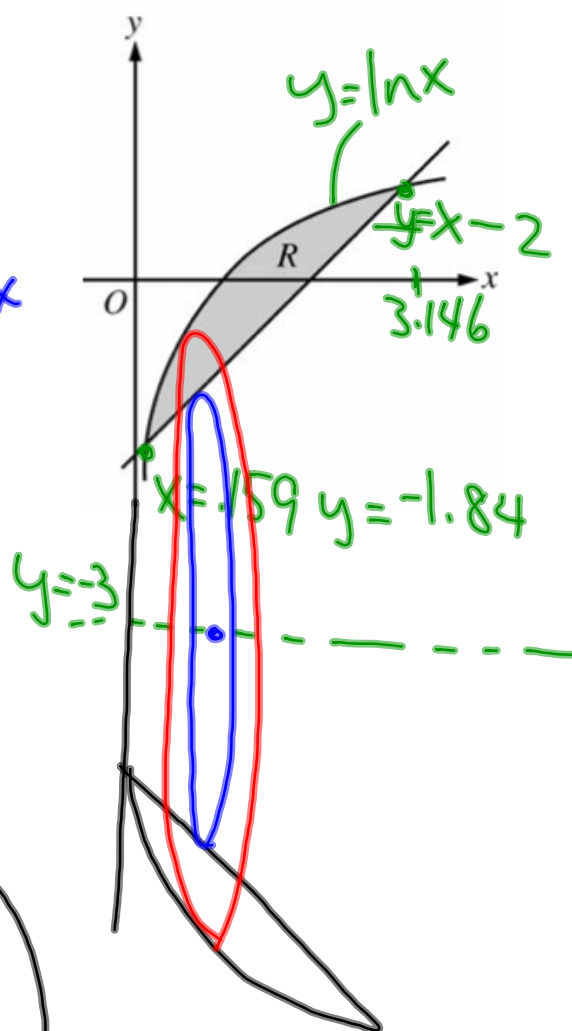
$= \boxed{.0777 \text{ u}^3}$

Let  $R$  be the shaded region bounded by the graph of  $y = \ln x$  and the line  $y = x - 2$ , as shown above.

- Find the area of  $R$ .
- Find the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = -3$ .
- Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when  $R$  is rotated about the  $y$ -axis.

$$a) A = \int_{.159}^{3.146} (\ln x - (x-2)) dx = 1.949 \text{ u}^2$$

$$b) V = \pi \int_{.159}^{3.146} (\ln x + 3)^2 - (x-2+3)^2 dx = 34.2 \text{ u}^3$$



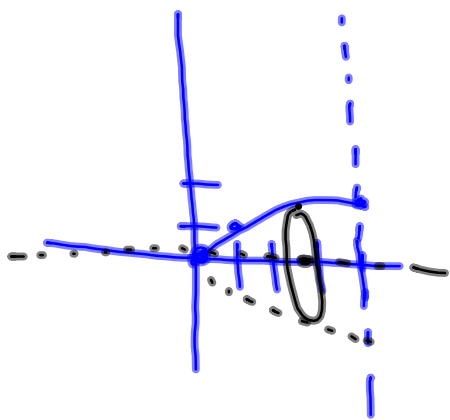
c)

$$V = \pi \int_{-1.84}^{1.146} (y+2)^2 - (e^y)^2 dy$$

The region bounded by the curve  $y = \sqrt{x}$  the  $x$ -axis and the line  $x = 4$  is revolved about the  $x$ -axis. Find the volume of the solid.

$$A = \pi r^2$$

Use discs



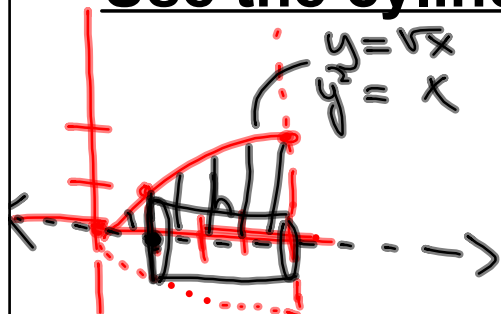
$$V = \pi \int_0^4 (\sqrt{x})^2 dx$$

$$= \pi \cdot \frac{x^2}{2} \Big|_0^4 = \frac{\pi \cdot 16}{2} - 0 = 8\pi$$

The region bounded by the curve  $y = \sqrt{x}$  the  $x$ -axis and the line  $x = 4$  is revolved about the  $x$ -axis. Find the volume of the solid.

$$\rightarrow 2\pi r h$$

**Use the cylindrical shell method**



Shell = parallel to the revolving axis

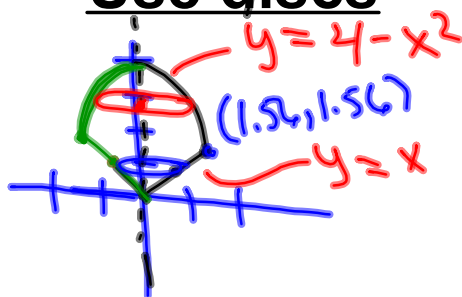
$$V = \int_0^2 2\pi r \cdot h \, dy$$

$$= \int_0^2 2\pi y(4 - y^2) \, dy$$

$$= 8\pi$$

The region bounded by the curve  $y = 4 - x^2$ ,  $y = x$ , and  $x = 0$  is revolved around the y-axis to form a solid. Find the volume of the solid.

**Use discs**



$$x^2 = 4 - y$$

$$x = \sqrt{4 - y}$$

$$4 - x^2 = x$$

$$x = 1.56$$

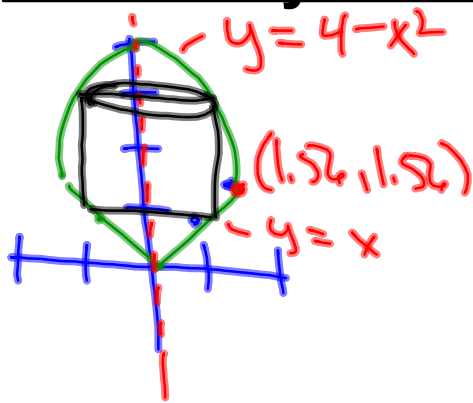
$$V = \pi \int_{1.56}^4 (\sqrt{4-y})^2 dy = 9.35$$

$$\oplus \pi \int_0^{1.56} (y)^2 dy + 3.97$$

$$\boxed{13.33}$$

The region bounded by the curve  $y = 4 - x^2$ ,  $y = x$  and  $x = 0$  is revolved around the  $y$ -axis to form a solid. Find the volume of the solid.

**Use the cylindrical shell method**



$2\pi r h$

parallel w/  
revolving  
Axis

$$V = \int_0^{1.52} 2\pi x (4 - x^2 - x) dx$$

$$= 13.3$$

# 55-61

revolving  
around  
 $y$

$$S = \int_a^b 2\pi (g(y)) \sqrt{1 + (g'(y))^2} dy$$

revolving  
around  
 $x$

$$S = \int_a^b 2\pi g(x) \sqrt{1 + (g'(x))^2} dx$$