AP Calculus AB
Worksheet 7.3

Name Key

In the following problems, set up integrals for finding the volume of revolution using shell methods.

1. Find the volume of the solid generated by revolving the region enclosed by the parabola $y^2 = 4x$ and the line $y = x$ about:

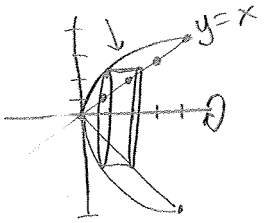
$x = \frac{y^2}{4}$

a. the x-axis

$V = 2\pi r h$

$h = y - \frac{y^2}{4}$

$r = y$



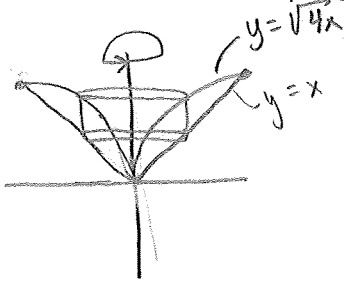
$$V = 2\pi \int_0^4 y\left(y - y^2/4\right) dy$$

b. the y-axis

$V = 2\pi r h$

$r = x$

$h = \sqrt{4x} - x$



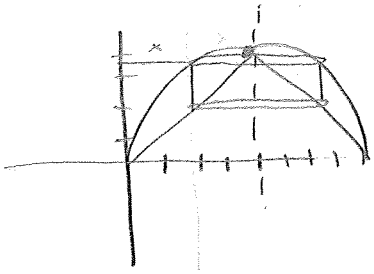
$$V = 2\pi \int_0^4 x\left(\sqrt{4x} - x\right) dx$$

c. the line x = 4

$V = 2\pi r h$

$r = 4 - x$

$h = \sqrt{4x} - x$



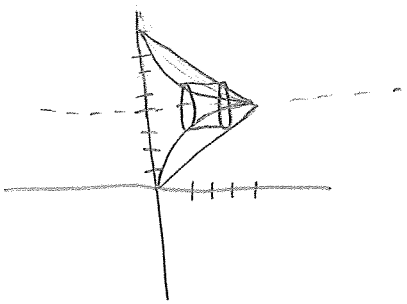
$$V = 2\pi \int_0^4 (4-x)\left(\sqrt{4x} - x\right) dx$$

d. the line y = 4

$V = 2\pi r h$

$r = 4 - y$

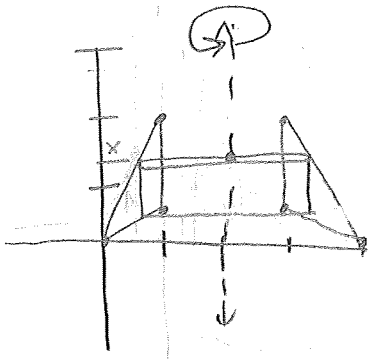
$h = y - \frac{y^2}{4}$



$$V = 2\pi \int_0^4 (4-y)\left(y - y^2/4\right) dy$$

2. Use the cylindrical shell method to find the volume of the solid generated by revolving the region bounded by:

a. $y = 2x$, $y = \frac{x}{2}$, and $x = 1$ about the line $x = 2$



$V = 2\pi r h$

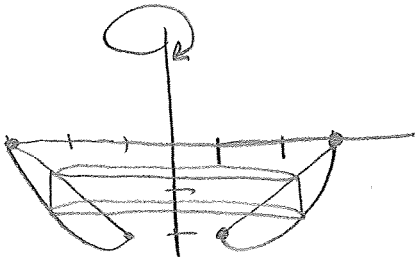
$r = 2 - x$

$h = 2x - \frac{x}{2} = \frac{3x}{2}$

$$V = 2\pi \int_0^1 (2-x)\left(\frac{3x}{2}\right) dx$$

$$= \boxed{2\pi}$$

b. $y = x - 3$, $y = x^2 - 3x$ about the y-axis



$V = 2\pi r h$

$r = x$

$h = x - 3 - (x^2 - 3x)$

$h = x - 3 - x^2 + 3x$

$h = -x^2 + 4x - 3$

$$V = 2\pi \int_1^3 x(-x^2 + 4x - 3)\, dx$$

$$= \boxed{\frac{16\pi}{3}}$$