AP-Calc Multiple Choice Practice Name Key

1. The average value of the function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is

(A) $-\frac{16}{3}$ (B) $\frac{16}{3}$ (C) $\frac{64}{3}$ (D) $\frac{66}{3}$ (E) $\frac{256}{3}$

$$\frac{1}{5-1}\int_1^5 (x-1)^2\, dx = \frac{1}{4}\cdot\int_1^5 (x^2-2x+1)\,dx$$
$$= \frac{1}{4}\left(\frac{x^3}{3} - x^2 + x\right)\Big|_1^5 = \frac{1}{4}\left(\frac{5^3}{3} - 25 + 5 - \left(\frac{1}{3} - 1 + 1\right)\right)$$
$$= \boxed{5\tfrac{1}{3}}$$

2. The average value of the function $f(x) = \ln^2 x$ on the interval [2, 4] is

(A) −1.204 (B) 1.204 (C) 2.159 (D) 2.408 (E) 8.636

$$\frac{1}{4-2}\int_2^4 \ln^2 x \cdot dx = (2.408)\frac{1}{2} = \boxed{1.204}$$

$$.29289 + .7071 = .9999 = 1$$

3. $\int_0^{\frac{\pi}{4}} \sin x\, dx + \int_{-\frac{\pi}{4}}^{0} \cos x\, dx =$

(A) $-\sqrt{2}$ (B) −1 (C) 0 (D) 1 (E) $\sqrt{2}$

$$-\cos x\Big|_0^{\pi/4} + \sin x\Big|_{-\pi/4}^{0} = -\frac{\sqrt{2}}{2} + 1 + 0 - -\frac{\sqrt{2}}{2} = 1$$

4. If $x^2 - 2xy + 3y^2 = 8$, then $\frac{dy}{dx} =$

(A) $\frac{8+2y-2x}{6y-2x}$

(B) $\frac{3y-x}{y-x}$

(C) $\frac{2x-2y}{6y-2x}$

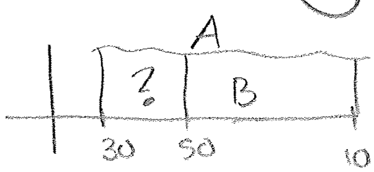
(D) $\frac{1}{3}$

(E) $\frac{y-x}{3y-x}$

$$2x - 2x\cdot\frac{dy}{dx} - 2y + 6y\cdot\frac{dy}{dx} = 0$$
$$2x - 2y = 2x\cdot\frac{dy}{dx} - 6y\cdot\frac{dy}{dx}$$
$$2x-2y = \frac{dy}{dx}(2x-6y)$$
$$\frac{dy}{dx} = \frac{2x-2y}{2x-6y} = \frac{x-y}{x-3y} = \frac{y-x}{3y-x}$$

5. If $\int_{30}^{100} f(x)\,dx = A$ and $\int_{50}^{100} f(x)\,dx = B$, then $\int_{30}^{50} f(x)\,dx =$

(A) $A + B$ (B) $A - B$ (C) 0 (D) $B - A$ (E) 20



A - B = ?

$(3x^2+2x)^{1/2}$

6. An equation of the line normal to the graph of $y = \sqrt{(3x^2 + 2x)}$ at (2, 4) is

(A) $-4x + y = 20$ (B) $4x + 7y = 20$ (C) $-7x + 4y = 2$ (D) $7x + 4y = 30$ (E) $4x + 7y = 36$

$y' = \frac{1}{2}(3x^2+2x)^{-1/2}(6x+2)$

$y'(2) = \frac{1}{2}(3\cdot 4+4)^{-1/2}(12+2) = \frac{1}{2}\cdot\frac{1}{\sqrt{16}}\cdot 14 = \frac{7}{4} = m$

$-\frac{4}{7} = m$

$y-4 = -\frac{4}{7}(x-2)$

$7y - 28 = -4x + 8$

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7. $\int_{-1}^{1} \frac{4}{1+x^2}\,dx =$ $4\int_{-1}^{1} \frac{1}{1+x^2}\cdot dx = 4\cdot \tan^{-1}x \Big|_{-1}^{1} = 4(\tan^{-1}(1) - \tan^{-1}(-1)) = 4\cdot(\frac{\pi}{4} - -\frac{\pi}{4})$

$= 4\left(\frac{2\pi}{4}\right)$

$= \boxed{2\pi}$

(A) 0 (B) π (C) 1 (D) 2π (E) 2

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8. If $f(x) = \cos^2 x$, then $f''(\pi) =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 2π

$f'(x) = -2\cos x \sin x$

$f''(x) = -2\cos x \cdot \cos x + 2\sin x \cdot \sin x = -2\cos^2 x + 2\sin^2 x$

$= -2\cos^2\pi + 2\sin^2\pi = -2\cdot 1 + 2\cdot 0 =$

$= \boxed{-2}$

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9. If $f(x) = \frac{5}{x^2+1}$ and $g(x) = 3x$, then $g(f(2)) =$ $g(1) = 3\cdot 1 = 3$

$\frac{5}{2^2+1} = \frac{5}{5} = 1$

(A) -3 (B) $\frac{5}{37}$ (C) 3 (D) 5 (E) $\frac{37}{5}$

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10. If $f(x) = \sec x + \csc x$, then $f'(x) =$

$f' = \sec x \tan x - \csc x \cot x$

(A) 0
(B) $\sec^2 x + \csc^2 x$
(C) $\csc x - \sec x$
(D) $\sec x \tan x + \csc x \cot x$
(E) $\sec x \tan x - \csc x \cot x$