Quiz 3-A, Sections 3.1 - 3.4 Solutions

1. $\lim_{h\to 0} \frac{3(x+h)^2+4-(3x^2+4)}{h} = \lim_{h\to 0} \frac{3x^2+6xh+3h^2+4-3x^2-4}{h}$

$= \lim_{h\to 0} \frac{6xh+3h^2}{h} = \lim_{h\to 0} 6x+3h = 6x$ $\boxed{f'(x)=6x}$

2. $\lim_{h\to 0} \frac{\frac{1}{x+h-2}-\frac{1}{x-2}}{h} = \lim_{h\to 0} \frac{\frac{x-2-(x+h-2)}{(x+h-2)(x+2)}}{h} = \lim_{h\to 0} \frac{\frac{x-2-x-h+2}{(x+h-2)(x+2)}}{h}$

$\lim_{h\to 0} \frac{-h}{(x+h-2)(x-2)} \cdot \frac{1}{h} = \lim_{h\to 0} \frac{-1}{(x+h-2)(x+2)} = \frac{-1}{(x-2)(x-2)}$

$f'(4) = \frac{-1}{(4-2)(4-2)} = \frac{-1}{4}$ $\quad y = \frac{1}{4-2} = \frac{1}{2}$ $\quad (4, \frac{1}{2})$

$\boxed{y-\frac{1}{2} = \frac{-1}{4}(x-4) \text{ or } y = \frac{-x}{4}+\frac{3}{2}}$

3. $y = -x^2+3x-2 \quad x=2$

$y' = -2x+3$

$y'(2) = -2(2)+3 = -4+3 = \boxed{-1}$

4. Math #8 nDeriv$((2x^7-4x^3)/(3x^2\cos(x^3)), x, .5)$

$= \boxed{-3.231}$

5. $y = 5x^4 - 2x^3 + 3x^2 + 6$

$\frac{dy}{dx} = 20x^3 - 6x^2 + 6x$

6. $y = \frac{-1}{2}x^3 + 2x \qquad y' = \frac{-3}{2}x^2 + 2 \qquad y'' = -3x$

7. $y = (x-3)(x^2+2)$ (u, v)

$y' = (x-3)(2x) + (1)(x^2+2) = 2x^2 - 6x + x^2 + 2 = 3x^2 - 6x + 2$

8. $y = \frac{3x+2}{x-4}$ (u, v) $\qquad y' = \frac{(x-4)(3) - 1(3x+2)}{(x-4)^2} = \frac{3x-12-3x-2}{(x-4)^2}$

$y' = \frac{-14}{(x-4)^2}$

9. $S = 10 + 80t - 16t^2$

$S' = v = 80 - 32t = 0$

$80 = 32t$

$2.5 = t$

2.5 seconds

10. $g = 2(15-t)^2 = 2(15-t)(15-t) = 450 - 60t + 2t^2 = g$

$-60 + 4t = g'$

$-60 + 4(6) = g'(6)$

$-36 = g'(6)$

-36 gal/min or leaking at 36 gal/min

Quiz 3-B, Sections 3.5 + 3.8, Solutions

1. $y = 3x^3 - \cot x$ $\quad \boxed{y' = 9x^2 + \csc^2 x}$

2. $y = \dfrac{\overset{u}{2x}}{\underset{v}{1+\sec x}}$ $\quad y' = \dfrac{(1+\sec x)(2) - 2x(\sec x \tan x)}{(1+\sec x)^2}$

$$\boxed{y' = \frac{2 + 2\sec x - 2x \sec x \tan x}{(1+\sec x)^2}}$$

3. $y = \overset{u}{x} \overset{v}{\sin}(2-x^2)$ $\quad y' = 1 \cdot \sin(2-x^2) + x\cos(2-x^2)\cdot -2x$

$$\boxed{y' = \sin(2-x^2) + -2x^2\cos(2-x^2)}$$

4. $y = \left(\dfrac{x^2}{3} - x\right)^{-2/3}$ $\quad y' = \dfrac{-2}{3}\left(\dfrac{x^2}{3} - x\right)^{-\frac{5}{3}} \cdot \left(\dfrac{2}{3}x - 1\right)$

$$\boxed{y' = \frac{-2}{3\left(\frac{x^2}{3} - x\right)^{5/3}} \cdot \left(\frac{2}{3}x - 1\right)}$$

5. $y = \cos^2(\sin 2x)$ or $y = (\cos(\sin 2x))^2$ $\quad y' = 2\cos(\sin(2x)) \cdot -\sin(\sin(2x)) \cdot \cos(2x) \cdot 2$

$$\boxed{y' = -4\cos(\sin(2x)) \cdot \sin(\sin(2x))\cos(2x)}$$

6. $y = \overset{u}{-3x}\overset{v}{\sqrt{x+6}}$ $\quad y' = -3\sqrt{x+6} + -3x \cdot \frac{1}{2}(x+6)^{-\frac{1}{2}} \cdot 1$

$$\boxed{y' = -3\sqrt{x+6} - \frac{3x}{2\sqrt{x+6}}}$$

7. $\overset{u}{2x}\overset{v}{y} - x^2 = y^2 + 2x$

$$2x \cdot \frac{dy}{dx} + 2 \cdot y - 2x = 2y\frac{dy}{dx} + 2$$

$$2x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2 - 2y + 2x$$

$$\frac{dy}{dx}(2x - 2y) = 2 - 2y + 2x$$

$$\frac{dy}{dx} = \frac{2 - 2y + 2x}{2x - 2y} = \boxed{\frac{1 - y + x}{x - y}}$$

8. $\overset{u}{x^2}\overset{v}{y^{3/2}} = 3$

$$2xy^{3/2} + x^2 \cdot \frac{3}{2}y^{1/2} \cdot \frac{dy}{dx} = 0 \qquad \left(\frac{1}{3}, 9\right)$$

$$\frac{dy}{dx}\left(x^2 \cdot \frac{3}{2} \cdot y^{1/2}\right) = -2xy^{3/2}$$

$$\frac{dy}{dx} = \frac{-2xy^{3/2}}{x^2 \cdot \frac{3}{2} \cdot y^{1/2}} = \frac{-2\left(\frac{1}{3}\right)(9)^{3/2}}{\left(\frac{1}{3}\right)^2\left(\frac{3}{2}\right)9^{1/2}} = \frac{-18}{\frac{1}{2}} = -36$$

$$\boxed{y - 9 = -36\left(x - \frac{1}{3}\right) \text{ OR } y = -36x + 21}$$

9. $L(x)$ $\quad f(x)=\sqrt{x-2}$ $\quad$ at $x=5$

10. $y = 2\sec\frac{x}{2}$

$$\frac{dy}{dx} = 2\sec\frac{x}{2}\cdot\tan\frac{x}{2}\cdot\frac{1}{2}$$

$$\frac{dy}{dx} = \sec\frac{x}{2}\tan\frac{x}{2}$$

$$dy = \sec\frac{x}{2}\tan\frac{x}{2}\,dx$$