Chapter 4 – AP Calc MC Questions (Derivative Applications)

FIRST DERIVATIVE FUNCTION ANALYSIS

7. For what value of k will $x+\frac{k}{x}$ have a relative maximum at $x=-2$?

D

(A) -4 (B) -2 (C) 2 (D) 4 (E) None of these

---

30. If a function f is continuous for all x and if f has a relative maximum at $(-1,4)$ and a relative minimum at $(3,-2)$, which of the following statements must be true?

E

(A) The graph of f has a point of inflection somewhere between $x=-1$ and $x=3$.
(B) $f'(-1)=0$
(C) The graph of f has a horizontal asymptote.
(D) The graph of f has a horizontal tangent line at $x=3$.
(E) The graph of f intersects both axes.

---

10. The *derivative* of $f(x)=\frac{x^4}{3}-\frac{x^5}{5}$ attains its maximum value at $x=$

C

(A) -1 (B) 0 (C) 1 (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

---

15. For what value of x does the function $f(x)=(x-2)(x-3)^2$ have a relative maximum?

D

(A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

---

27. If $f(x)=\frac{1}{3}x^3-4x^2+12x-5$ and the domain is the set of all x such that $0\le x\le 9$, then the absolute maximum value of the function f occurs when x is

E

(A) 0 (B) 2 (C) 4 (D) 6 (E) 9

---

2. At what values of x does $f(x)=3x^5-5x^3+15$ have a relative maximum?

A

(A) -1 only (B) 0 only (C) 1 only (D) -1 and 1 only (E) -1, 0 and 1

---

16. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers x has a relative maximum at $x =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4

B

---

33. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2

A

---

5. The function f given by $f(x) = 2x^3 - 3x^2 - 12x$ has a relative minimum at $x =$

(A) -1 (B) 0 (C) 2 (D) $\frac{3 - \sqrt{105}}{4}$ (E) $\frac{3 + \sqrt{105}}{4}$

C

---

44. What is the minimum value of $f(x) = x \ln x$?

(A) $-e$ (B) -1 (C) $-\frac{1}{e}$ (D) 0 (E) $f(x)$ has no minimum value.

C

---

85. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

(A) -0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

C

---

92. Let f be the function with first derivative defined by $f'(x) = \sin(x^3)$ for $0 \le x \le 2$. At what value of x does f attain its maximum value on the closed interval $0 \le x \le 2$?

(A) 0 (B) 1.162 (C) 1.465 (D) 1.845 (E) 2

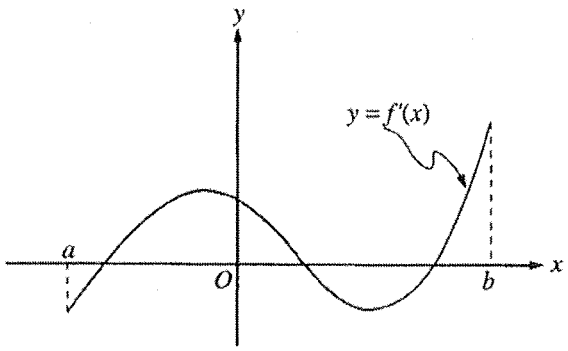
C

---

3. The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ has a relative maximum at $x =$

(A) -1 (B) $-\frac{\sqrt{5}}{5}$ (C) 0 (D) $\frac{\sqrt{5}}{5}$ (E) 1

A



A

12. The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a,b)?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

---

14. The derivative of f is $x^4(x-2)(x+3)$. At how many points will the graph of f have a relative maximum?

B

(A) None (B) One (C) Two (D) Three (E) Four

---

36. If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value of $f(x)$ is -7, then which of the following must be true?

I. The maximum value of $f(|x|)$ is 5.
II. The maximum value of $|f(x)|$ is 7.
III. The minimum value of $f(|x|)$ is 0.

B

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

---

24. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

D

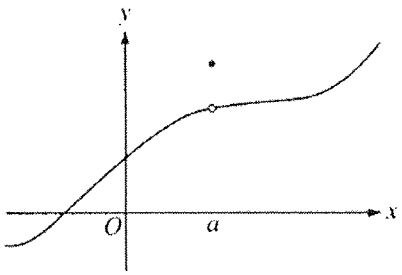
(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

---

19. A polynomial $p(x)$ has a relative maximum at $(-2,4)$, a relative minimum at $(1,1)$, a relative maximum at $(5,7)$ and no other critical points. How many zeros does $p(x)$ have?

B

(A) One (B) Two (C) Three (D) Four (E) Five



76. The graph of a function f is shown above. Which of the following statements about f is false?

A

(A) f is continuous at $x = a$.

(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x \to a^+} f(x)$ is equal to $\lim_{x \to a^-} f(x)$.

(E) $\lim_{x \to a} f(x)$ exists.

---

19. A point moves on the x-axis in such a way that its velocity at time t $(t > 0)$ is given by $v = \dfrac{\ln t}{t}$. At what value of t does v attain its maximum?

C

(A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for v.

---

80. The first derivative of the function f is given by $f'(x) = \dfrac{\cos^2 x}{x} - \dfrac{1}{5}$. How many critical values does f have on the open interval $(0,10)$?

B

(A) One
(B) Three
(C) Four
(D) Five
(E) Seven

---

81. Let f be the function with derivative given by $f'(x) = \sin(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?

D

(A) One (B) Two (C) Three (D) Four (E) Five

---

81. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

D

I. f is continuous at $x = 0$.
II. f is differentiable at $x = 0$.
III. f has an absolute minimum at $x = 0$.

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

---

22. If $f'(x) = (x-2)(x-3)^2(x-4)^3$, then f has which of the following relative extrema?

A

I. A relative maximum at $x = 2$
II. A relative minimum at $x = 3$
III. A relative maximum at $x = 4$

(A) I only
(B) III only
(C) I and III only
(D) II and III only
(E) I, II, and III

---

89. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = \left(x^2 - 4\right)g(x)$, which of the following is true?

B

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if f has any relative extrema.

---

23. Let f be a function defined and continuous on the closed interval $[a,b]$. If f has a relative maximum at c and $a < c < b$, which of the following statements must be true?

E

I. $f'(c)$ exists.
II. If $f'(c)$ exists, then $f'(c) = 0$.
III. If $f''(c)$ exists, then $f''(c) \le 0$.

(A) II only (B) III only (C) I and II only (D) I and III only (E) II and III only

---

21. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

B

---

3. If $f(x) = x + \frac{1}{x}$, then the set of values for which f increases is

(A) $(-\infty, -1] \cup [1, \infty)$ (B) $[-1, 1]$ (C) $(-\infty, \infty)$

(D) $(0, \infty)$ (E) $(-\infty, 0) \cup (0, \infty)$

A

---

22. What are all values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

(A) There are no such values of x.
(B) $x < -1$ and $x > 3$
(C) $-3 < x < 1$
(D) $-1 < x < 3$
(E) All values of x

D

---

1. What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

(A) $-3 < x < 1$
(B) $-1 < x < 1$
(C) $x < -3$ or $x > 1$
(D) $x < -1$ or $x > 3$
(E) All real numbers

C

---

22. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

(D) $(-\infty, 0)$

(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

C

---

15. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

(A) $(-\infty, -1]$ only

(B) $(-\infty, 0)$

(C) $[-1, 0)$ only

(D) $(0, \sqrt[3]{2}]$

(E) $[\sqrt[3]{2}, \infty)$

D

---

x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

18. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

(A) $-2 \le x \le 2$ only

(B) $-1 \le x \le 1$ only

(C) $x \ge -2$

(D) $x \ge 2$ only

(E) $x \le -2$ or $x \ge 2$

A

---

78. The first derivative of the function f is defined by $f'(x) = \sin(x^3 - x)$ for $0 \le x \le 2$. On what interval(s) is f increasing?

(A) $1 \le x \le 1.445$

(B) $1 \le x \le 1.691$

(C) $1.445 \le x \le 1.875$

(D) $0.577 \le x \le 1.445$ and $1.875 \le x \le 2$

(E) $0 \le x \le 1$ and $1.691 \le x \le 2$

B

---

27. The function f given by $f(x) = x^3 + 12x - 24$ is

(A) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
(B) decreasing for $x < 0$, increasing for $x > 0$
(C) increasing for all x
(D) decreasing for all x
(E) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

C

---

22. If $f(x) = x^2 e^x$, then the graph of f is decreasing for all x such that

(A) $x < -2$ (B) $-2 < x < 0$ (C) $x > -2$ (D) $x < 0$ (E) $x > 0$

B

---

13. Let f be a function defined for all real numbers x. If $f'(x) = \dfrac{|4 - x^2|}{x - 2}$, then f is decreasing on the interval

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$

A

---

9. If $f(x) = 1 + x^{\frac{2}{3}}$, which of the following is NOT true?

(A) f is continuous for all real numbers.
(B) f has a minimum at $x = 0$.
(C) f is increasing for $x > 0$.
(D) $f'(x)$ exists for all x.
(E) $f''(x)$ is negative for $x > 0$.

D

---

19. Let f be the function defined by $f(x)=\begin{cases} x^3 & \text{for } x \le 0, \\ x & \text{for } x > 0. \end{cases}$ Which of the following statements about f is true?

(A) f is an odd function.

(B) f is discontinuous at $x=0$.

(C) f has a relative maximum.

(D) $f'(0)=0$

(E) $f'(x)>0$ for $x \neq 0$

E

---

39. If $f(x)=\dfrac{\ln x}{x}$, for all $x>0$, which of the following is true?

(A) f is increasing for all x greater than 0.

(B) f is increasing for all x greater than 1.

(C) f is decreasing for all x between 0 and 1.

(D) f is decreasing for all x between 1 and e.

(E) f is decreasing for all x greater than e.

E

---

23. How many critical points does the function $f(x)=(x+2)^5(x-3)^4$ have?

(A) One (B) Two (C) Three (D) Five (E) Nine

C

---

45. If f is a continuous function on $[a,b]$, which of the following is necessarily true?

(A) f' exists on (a,b).

(B) If $f(x_0)$ is a maximum of f, then $f'(x_0)=0$.

(C) $\lim\limits_{x \to x_0} f(x) = f\left(\lim\limits_{x \to x_0} x\right)$ for $x_0 \in (a,b)$

(D) $f'(x)=0$ for some $x \in [a,b]$

(E) The graph of f' is a straight line.

C

SECOND DERIVATIVE FUNCTION ANALYSIS

80. Let f be the function given by $f(x)=\cos(2x)+\ln(3x)$. What is the least value of x at which the graph of f changes concavity? **B**

(A) 0.56 (B) 0.93 (C) 1.18 (D) 2.38 (E) 2.44

---

1. What is the x-coordinate of the point of inflection on the graph of $y=\frac{1}{3}x^3+5x^2+24$? **D**

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10

---

77. The graph of the function $y=x^3+6x^2+7x-2\cos x$ changes concavity at $x=$ **D**

(A) -1.58 (B) -1.63 (C) -1.67 (D) -1.89 (E) -2.33

---

43. An equation of the line tangent to $y=x^3+3x^2+2$ at its point of inflection is **B**

(A) $y=-6x-6$ (B) $y=-3x+1$ (C) $y=2x+10$

(D) $y=3x-1$ (E) $y=4x+1$

---

21. At what value of x does the graph of $y=\frac{1}{x^2}-\frac{1}{x^3}$ have a point of inflection? **C**

(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of x

---

17. The graph of $y=5x^4-x^5$ has a point of inflection at **B**

(A) (0,0) only (B) (3,162) only (C) (4,256) only

(D) (0,0) and (3,162) (E) (0,0) and (4,256)

---

19. If $f''(x)=x(x+1)(x-2)^2$, then the graph of f has inflection points when $x=$ **C**

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

---

16. If f is the function defined by $f(x) = 3x^5 - 5x^4$, what are all the x-coordinates of points of inflection for the graph of f?

C

(A) -1 (B) 0 (C) 1 (D) 0 and 1 (E) -1, 0, and 1

---

87. The function f has first derivative given by $f'(x) = \dfrac{\sqrt{x}}{1 + x + x^3}$. What is the x-coordinate of the inflection point of the graph of f ?

B

(A) 1.008 (B) 0.473 (C) 0 (D) -0.278 (E) The graph of f has no inflection point.

---

20. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x-coordinates of the points of inflection of the graph of f?

D

(A) 0 only

(B) 3 only

(C) 0 and 6 only

(D) 3 and 6 only

(E) 0, 3, and 6

---

80. The derivative of the function f is given by $f'(x) = x^2 \cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

E

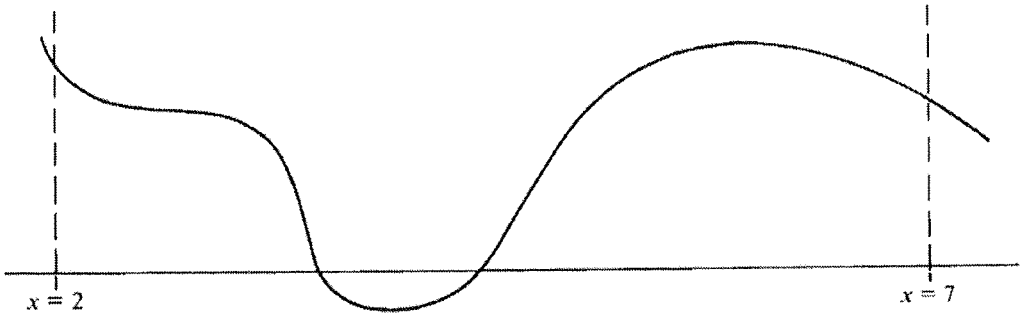
(A) One (B) Two (C) Three (D) Four (E) Five

---

81. The first derivative of the function f is given by $f'(x) = x - 4e^{-\sin(2x)}$. How many points of inflection does the graph of f have on the interval $0 < x < 2\pi$?

B

(A) Three (B) Four (C) Five (D) Six (E) Seven



C

20. The graph of $y = f(x)$ on the closed interval $[2, 7]$ is shown above. How many points of inflection does this graph have on this interval?

(A) One (B) Two (C) Three (D) Four (E) Five

---

27. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b?

B

(A) -3 (B) 0 (C) 1 (D) 3

(E) It cannot be determined from the information given.

---

22. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

B

(A) $x > 0$

(B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$

(C) $-2 < x < 0$ or $x > 2$

(D) $x > \sqrt{2}$

(E) $-2 < x < 2$

---

4. The graph of $y = \dfrac{-5}{x-2}$ is concave downward for all values of x such that

E

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x > 2$

---

17. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

A

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

---

5. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

E

---

19. Let f be the function given by $f(x) = x^3 - 6x^2$. The graph of f is concave up when

(A) $x > 2$

(B) $x < 2$

(C) $0 < x < 4$

(D) $x < 0$ or $x > 4$ only

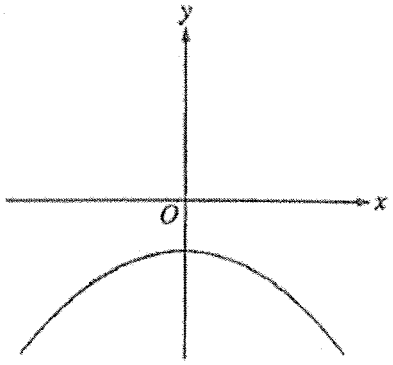
(E) $x > 6$ only

A

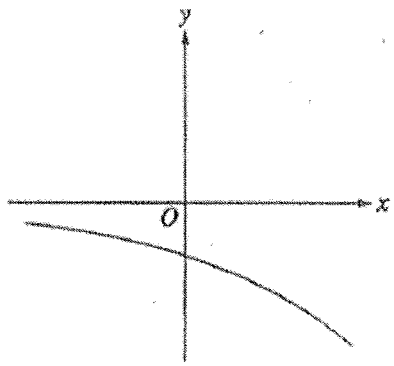
FIRST AND SECOND DERIVATIVE FUNCTION ANALYSIS

10. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values x. Which of the following could be the graph of f ?

(A)

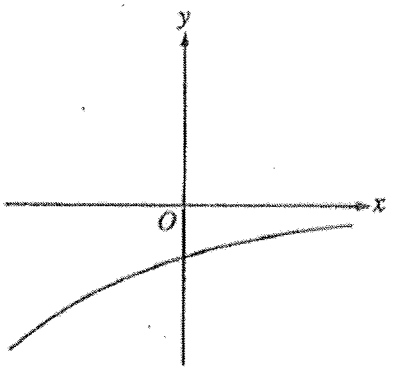


(B)

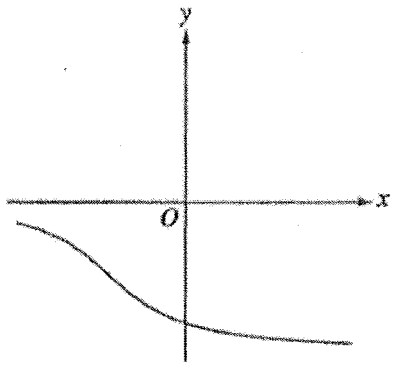


B

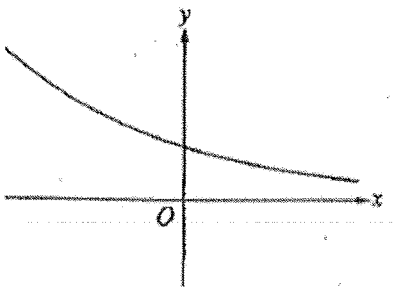
(C)



(D)



(E)



---

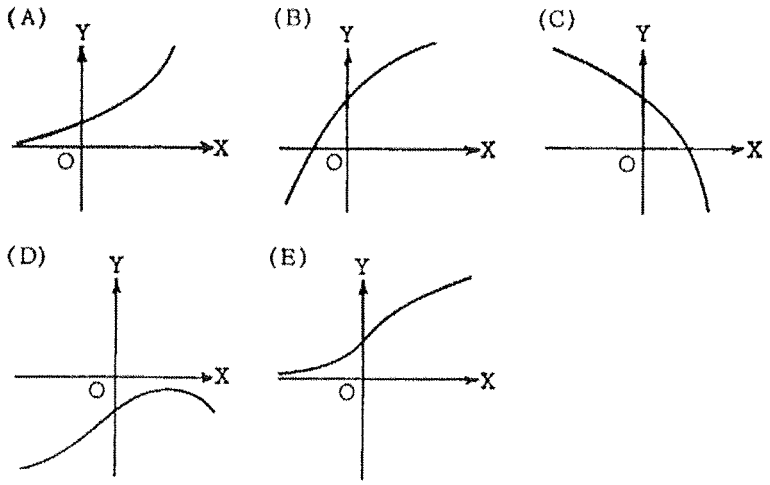
28. Let g be a twice-differentiable function with $g'(x) > 0$ and $g''(x) > 0$ for all real numbers x, such that $g(4) = 12$ and $g(5) = 18$. Of the following, which is a possible value for $g(6)$?

E

(A) 15 (B) 18 (C) 21 (D) 24 (E) 27

---

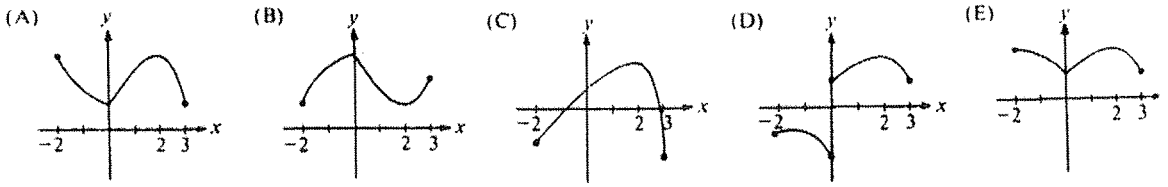
16. If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



B

---

43. Let f be a function that is continuous on the closed interval $[-2,3]$ such that $f'(0)$ does not exist, $f'(2) = 0$, and $f''(x) < 0$ for all x except $x = 0$. Which of the following could be the graph of f?



E

---

20. Let f be a polynomial function with degree greater than 2. If $a \neq b$ and $f(a) = f(b) = 1$, which of the following must be true for at least one value of x between a and b?

I. $f(x) = 0$
II. $f'(x) = 0$
III. $f''(x) = 0$

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

C

---

26. For $x > 0$, f is a function such that $f'(x) = \dfrac{\ln x}{x}$ and $f''(x) = \dfrac{1 - \ln x}{x^2}$. Which of the following is true?

(A) f is decreasing for $x > 1$, and the graph of f is concave down for $x > e$.

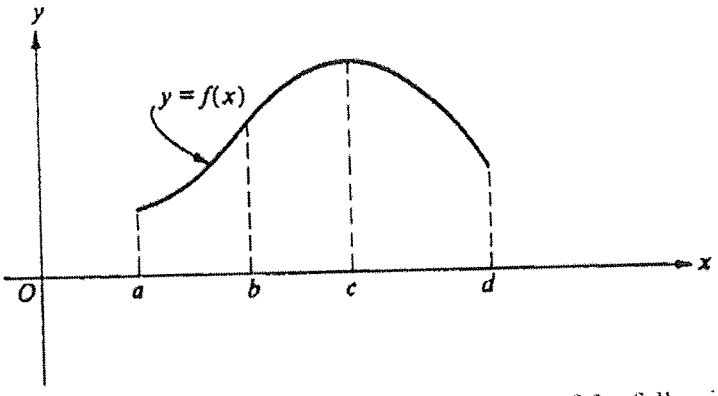
(B) f is decreasing for $x > 1$, and the graph of f is concave up for $x > e$.

(C) f is increasing for $x > 1$, and the graph of f is concave down for $x > e$.

(D) f is increasing for $x > 1$, and the graph of f is concave up for $x > e$.

(E) f is increasing for $0 < x < e$, and the graph of f is concave down for $0 < x < e^{3/2}$.

C

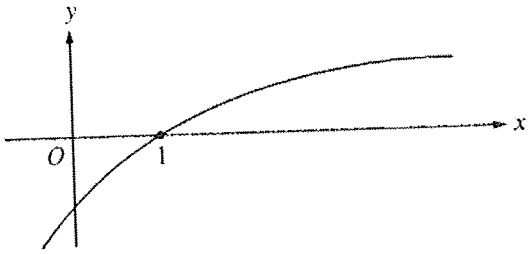


8. The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

B



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$
(B) $f(1) < f''(1) < f'(1)$
(C) $f'(1) < f(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f''(1) < f'(1) < f(1)$

D

90. For all x in the closed interval $[2, 5]$, the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

(A)

x	$f(x)$
2	7
3	9
4	12
5	16

(B)

x	$f(x)$
2	7
3	11
4	14
5	16

(C)

x	$f(x)$
2	16
3	12
4	9
5	7

(D)

x	$f(x)$
2	16
3	14
4	11
5	7

(E)

x	$f(x)$
2	16
3	13
4	10
5	7

B

x	0	1	2	3
$f''(x)$	5	0	-7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

E

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

---

90. The function f is continuous on the closed interval $[2, 4]$ and twice differentiable on the open interval $(2, 4)$. If $f'(3) = 2$ and $f''(x) < 0$ on the open interval $(2, 4)$, which of the following could be a table of values for f?

A

(A)

x	$f(x)$
2	2.5
3	5
4	6.5

(B)

x	$f(x)$
2	2.5
3	5
4	7

(C)

x	$f(x)$
2	3
3	5
4	6.5

(D)

x	$f(x)$
2	3
3	5
4	7

(E)

x	$f(x)$
2	3.5
3	5
4	7.5

MEAN VALUE THEOREM

3. The Mean Value Theorem guarantees the existence of a special point on the graph of $y=\sqrt{x}$ between $(0,0)$ and $(4,2)$. What are the coordinates of this point?

(A) $(2,1)$
(B) $(1,1)$
(C) $\left(2,\sqrt{2}\right)$
(D) $\left(\frac{1}{2},\frac{1}{\sqrt{2}}\right)$
(E) None of the above

B

---

13. Let f be the function given by $f(x)=x^3-3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0,3]$?

(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

B

---

24. If c is the number that satisfies the conclusion of the Mean Value Theorem for $f(x)=x^3-2x^2$ on the interval $0\le x\le 2$, then $c=$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{4}{3}$ (E) 2

D

---

18. If $f(x)=\sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2}<x<\frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) π (E) $\frac{3\pi}{2}$

D

OPTIMIZATION

11. The point on the curve $x^2 + 2y = 0$ that is nearest the point $\left(0, -\frac{1}{2}\right)$ occurs where y is

(A) $\frac{1}{2}$ (B) 0 (C) $-\frac{1}{2}$ (D) -1 (E) none of the above

B

---

39. The point on the curve $2y = x^2$ nearest to $(4,1)$ is

(A) $(0,0)$ (B) $(2,2)$ (C) $\left(\sqrt{2},1\right)$ (D) $\left(2\sqrt{2},4\right)$ (E) $(4,8)$

B

---

45. The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

(A) $2\sqrt[3]{2}$ (B) $2\sqrt{2}$ (C) $2\sqrt[3]{4}$ (D) 4 (E) 8

D

---

36. Consider all right circular cylinders for which the sum of the height and circumference is 30 centimeters. What is the radius of the one with maximum volume?

(A) 3 cm (B) 10 cm (C) 20 cm (D) $\frac{30}{\pi^2}$ cm (E) $\frac{10}{\pi}$ cm

E

---

82. If $y = 2x - 8$, what is the minimum value of the product xy ?

(A) -16 (B) -8 (C) -4 (D) 0 (E) 2

B

RELATED RATES

9. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is

C

(A) $\frac{1}{4\pi}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\pi}$ (D) 1 (E) π

---

24. The radius of a circle is increasing. At a certain instant, the rate of increase in the area of the circle is numerically equal to twice the rate of increase in its circumference. What is the radius of the circle at that instant?

D

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

---

22. The area of a circular region is increasing at a rate of 96π square meters per second. When the area of the region is 64π square meters, how fast, in meters per second, is the radius of the region increasing?

A

(A) 6 (B) 8 (C) 16 (D) $4\sqrt{3}$ (E) $12\sqrt{3}$

---

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)C}{2\pi}$

B

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$

---

39. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

D

(A) $\frac{1}{\pi}$ (B) $\frac{1}{2}$ (C) $\frac{2}{\pi}$ (D) 1 (E) 2

---

78. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

(A) 0.04π m^2/sec

(B) 0.4π m^2/sec

(C) 4π m^2/sec

(D) 20π m^2/sec

(E) 100π m^2/sec

C

---

34. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

(A) $-\frac{7}{8}$ feet per minute

(B) $-\frac{7}{24}$ feet per minute

(C) $\frac{7}{24}$ feet per minute

(D) $\frac{7}{8}$ feet per minute

(E) $\frac{21}{25}$ feet per minute

D

---

37. A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of $\frac{4}{9}$ meter per second, at what rate, in meters per second, is the person walking?

(A) $\frac{4}{27}$ (B) $\frac{4}{9}$ (C) $\frac{3}{4}$ (D) $\frac{4}{3}$ (E) $\frac{16}{9}$

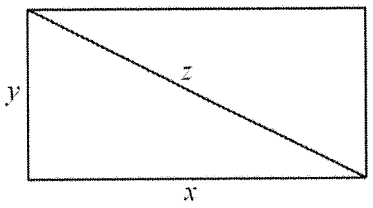
D

---

81. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40

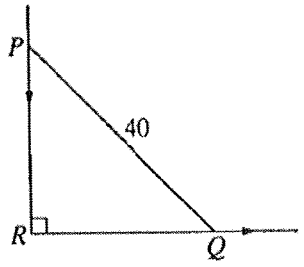
A



40. The sides of the rectangle above increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

B

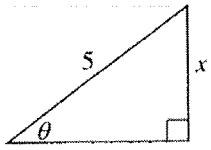
(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5



34. In the figure above, PQ represents a 40-foot ladder with end P against a vertical wall and end Q on level ground. If the ladder is slipping down the wall, what is the distance RQ at the instant when Q is moving along the ground $\frac{3}{4}$ as fast as P is moving down the wall?

E

(A) $\frac{6}{5}\sqrt{10}$ (B) $\frac{8}{5}\sqrt{10}$ (C) $\frac{80}{\sqrt{7}}$ (D) 24 (E) 32



23. In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

E

(A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12

---

88. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S = 4\pi r^2$)

C

(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π

---

26. The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3\right)$

E

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

---

31. The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

C

(A) $\frac{1}{2}\pi$ (B) 10π (C) 24π (D) 54π (E) 108π

---

4. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?

B

(A) $-\frac{5}{2}$ (B) $-\frac{6}{5}$ (C) 0 (D) $\frac{4}{5}$ (E) $\frac{6}{5}$

---

20. When $x = 8$, the rate at which $\sqrt[3]{x}$ is increasing is $\frac{1}{k}$ times the rate at which x is increasing. What is the value of k?

E

(A) 3 (B) 4 (C) 6 (D) 8 (E) 12

---

90. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

D

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

---

12. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) $V(t) = k\sqrt{t}$

(B) $V(t) = k\sqrt{V}$

(C) $\frac{dV}{dt} = k\sqrt{t}$

(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

(E) $\frac{dV}{dt} = k\sqrt{V}$

E

LINEARIZATIONS

36. The approximate value of $y=\sqrt{4+\sin x}$ at $x=0.12$, obtained from the tangent to the graph at $x=0$, is

B

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

---

44. For small values of h, the function $\sqrt[4]{16+h}$ is best approximated by which of the following?

(A) $4+\frac{h}{32}$ (B) $2+\frac{h}{32}$ (C) $\frac{h}{32}$

(D) $4-\frac{h}{32}$ (E) $2-\frac{h}{32}$

B

---

14. Let f be a differentiable function such that $f(3)=2$ and $f'(3)=5$. If the tangent line to the graph of f at $x=3$ is used to find an approximation to a zero of f, that approximation is

C

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

---

24. The function f is twice differentiable with $f(2)=1$, $f'(2)=4$, and $f''(2)=3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x=2$?

B

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

---

18. For the function f, $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$?

D

(A) 0.6 (B) 3.4 (C) 4.2 (D) 4.6 (E) 4.64
