CH. 4 WS #1 CALCULUS AB Name Key Per.______

1) $\int\left(x^7+\frac{3}{x^2}\right)dx = \int(x^7+3x^{-2})dx$

$\frac{x^8}{8}+\frac{3x^{-1}}{-1}+C = \boxed{\frac{x^8}{8}-\frac{3}{x}+C}$

2) $\int_8^{27} x^{-1/3}\,dx =$

$\frac{3x^{2/3}}{2}\Big|_8^{27} = \frac{3(27)^{2/3}}{2}-\frac{3(8)^{2/3}}{2} = \frac{27}{2}-\frac{12}{2} = \boxed{\frac{15}{2}}$

3) $\int 10x^4(2x^5+1)^7\,dx =$ $u=2x^5+1$, $du=10x^4\,dx$

$\int u^7\,du$

$\frac{u^8}{8}+C = \boxed{\frac{(2x^5+1)^8}{8}+C}$

4) $\int\frac{x^9}{(x^{10}-8)^{61}}\,dx =$ $u=x^{10}-8$, $du=10x^9\,dx$, $\frac{1}{10}du=x^9\,dx$

$\frac{1}{10}\int\frac{1}{u^{61}}\,du$

$\frac{1}{10}\cdot\frac{u^{-60}}{-60}+C = \boxed{\frac{1}{-600(x^{10}-8)^{60}}+C}$

5) $\int_0^1 2x\sqrt{-5x^2+9}\,dx =$ $u=-5x^2+9$, $du=-10x\,dx$, $-\frac{1}{5}du=2x\,dx$

$= -\frac{1}{5}\int u^{1/2}\,du$

$= -\frac{1}{5}u^{3/2}\cdot\frac{2}{3} = -\frac{2}{15}(-5x^2+9)^{3/2}\Big|_0^1 = -\frac{2}{15}(8-27)$

$= -\frac{16}{15}+\frac{54}{15} = \boxed{\frac{38}{15}}$

6) $\int\tan^7 t\cdot\sec^2 t\,dt =$ $u=\tan t$, $du=\sec^2 t\,dt$

$\int u^7\,du$

$\frac{u^8}{8}+C = \boxed{\frac{(\tan t)^8}{8}+C}$

7) $\int 2x\sqrt{x-6}\,dx =$

$u=x-6$
$du=1dx$
$u+6=x$
$2u+12=2x$

$= \int(2u+12)u^{1/2}\,du$
$= \int(2u^{3/2}+12u^{1/2})\,du$

$2u^{5/2}\cdot\frac{2}{5}+12u^{3/2}\cdot\frac{2}{3}+C$

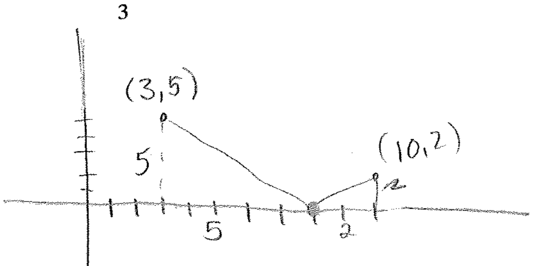
$= \boxed{\frac{4}{5}(x-6)^{5/2}+8(x-6)^{3/2}+C}$

8) $F(x)=\int_x^{x^3}\sqrt{t^5-2}\,dt$ Find $F'(x)$.

$\int_0^{x^3}\sqrt{t^5-2}\,dt-\int_0^{x}\sqrt{t^5-2}\,dt$

$\boxed{\sqrt{x^{15}-2}\cdot(3x^2)-\sqrt{x^5-2}}$

9) $\int_3^{10}|x-8|\,dx =$



$\frac{1}{2}\cdot5\cdot5+\frac{1}{2}\cdot2\cdot2 = \frac{25}{2}+2 = \boxed{\frac{29}{2}}$

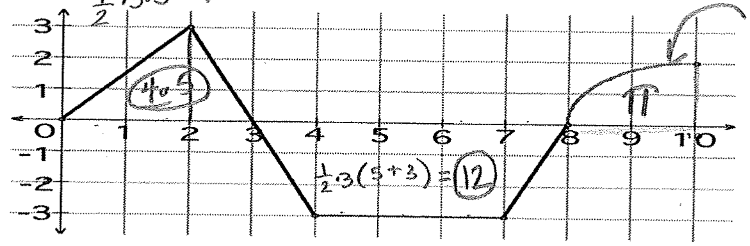
10) Find avg. value of $f(x)=9-x^2$ from $[0,2]$.

$= \frac{1}{2-0}\int_0^2(9-x^2)\,dx$

$\frac{1}{2}\left(9x-\frac{x^3}{3}\right)\Big|_0^2 = \frac{1}{2}\left(18-\frac{8}{3}-0\right)$

$= \frac{1}{2}\cdot\frac{46}{3} = \boxed{\frac{23}{3}}$

$\frac{1}{2}\cdot 3\cdot 3 = 4.5$



$A = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi 2^2 = \pi$

11) a) $\int_0^{10} f(x)\,dx = \boxed{-4.36}$ $\quad 4.5-12+\pi$

b) $\int_0^{10} |f(x)|\,dx = \boxed{19.64}$ $\quad 4.5+12+\pi$

c) $\int_0^{10} (f(x)+3)\,dx = \boxed{25.64}$ $\quad -4.36 + \int_0^{10} 3\,dx = -4.36 + 3x\Big|_0^{10} = -4.36+30$

d) $\int_7^2 f(x)\,dx = \boxed{-9}$ $\quad \frac{1}{2}\cdot 1\cdot 3 + -\left(\frac{1}{2}\cdot 3(4+3)\right)$

e) $6\int_8^{10} f(x)\,dx = \boxed{6\pi}$

f) $\int_0^4 f(x)\,dx = \boxed{3}$ $\quad 4.5-\frac{1}{2}\cdot 1\cdot 3 = 3$

g) $\int_8^4 f(x)\,dx = \boxed{10.5}$ $\quad -\int_4^8 f(x)\,dx = +\frac{1}{2}\cdot 3(4+3)$

h) Avg. value from $[0,3] =$ $\frac{1}{3-0}(4.5) = \frac{1}{3}\cdot\frac{9}{2} = \boxed{\frac{3}{2}}$

i) Avg. value from $[0,10] =$ $\frac{1}{10-0}\left[-4.36\right] = \boxed{-.436}$

12) To estimate the surface area of a pool, a surveyor takes several measurements. The measurements are taken every 10 feet for the 80 ft. long pond, where y represents the distance across the pool at each 10 ft. increment.

x	0	10	20	30	40	50	60	70	80
y	10	14	18	16	11	12	18	13	15

a) Estimate using Trapezoidal Rule

$\frac{1}{2}\cdot 10(10+2\cdot 14+2\cdot 18+2\cdot 16+2\cdot 11+2\cdot 12+2\cdot 18+2\cdot 13+15)$

$=\boxed{1145}$

b) Estimate Avg. value usingTrapezoidal Rule

$\frac{1}{80-0}(1145) = \boxed{14.3125}$

c) Estimate using Right Endpoint

$= 10(14+18+16+11+12+18+13+15)$

$=\boxed{1170}$

d) Estimate using 4 Midpoint subdivisions

$20\cdot 14 + 20\cdot 16 + 20\cdot 12 + 20\cdot 13$

$=\boxed{1100}$

13) To estimate the area of a plot of land, I took measurements as shown below right. The measurements are taken where y represents the distance across the land in feet at each increment. Approximate the area of the land.

a) Estimate using Left Endpoint

$4\cdot 8 + 2\cdot 14 + 5\cdot 10$

$\boxed{110}$

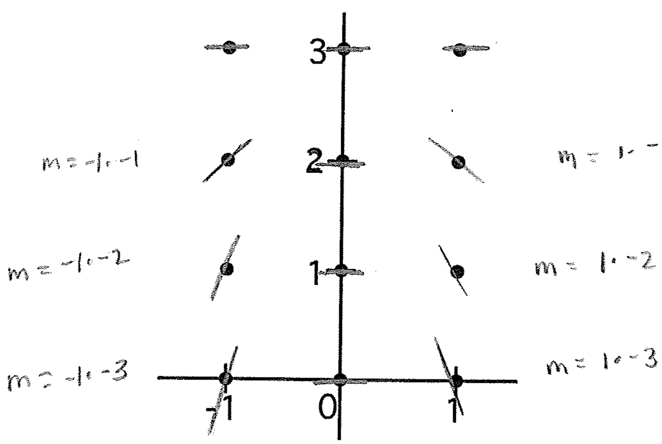
x	**0**	**4**	**6**	**11**
y	**8**	**14**	**10**	**9**

CH.6 Differential Equations / Slope Fields WS

Name____ Per.____

1) Given the differential equation $\frac{dy}{dx} = x(y-3)$.

a) Sketch a slope field for the given differential equation at the twelve points indicated.



b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 10$.

$\frac{dy}{dx} = x(y-3)$

$\int \frac{1}{y-3}\,dy = \int x\,dx$

$\ln(y-3) = \frac{x^2}{2} + C$

$\ln(10-3) = \frac{0^2}{2} + C$

$\ln 7 = C$

$\ln(y-3) = \frac{x^2}{2} + \ln 7$

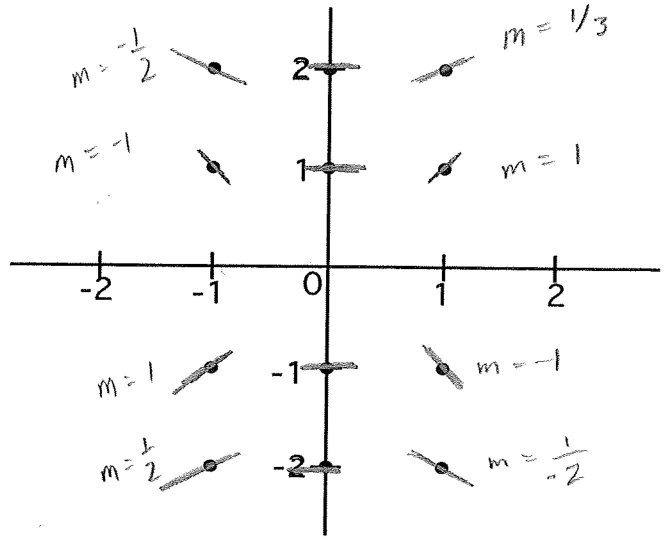
$e^{\frac{x^2}{2} + \ln 7} = y - 3$

$\boxed{y = e^{\frac{x^2}{2} + \ln 7} + 3}$

$\boxed{y = 7e^{x^2/2} + 3}$

2) Given the differential equation $\frac{dy}{dx} = \frac{x^3}{y}$.

a) Sketch a slope field for the given differential equation at the twelve points indicated.



b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 6$.

$\frac{dy}{dx} = \frac{x^3}{y}$

$\int y\,dy = \int x^3\,dx$

$\frac{y^2}{2} = \frac{x^4}{4} + C$

$\frac{6^2}{2} = \frac{2^4}{4} + C$

$18 = 4 + C$

$14 = C$

$\frac{y^2}{2} = \frac{x^4}{4} + 14$

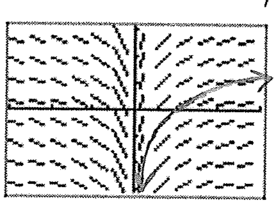
$y^2 = \frac{x^4}{2} + 28$

$y = \sqrt{\frac{x^4}{2} + 28}$

3a) Given $\frac{dy}{dx} = \frac{1}{x}$

Sketch the solution curve through the point (2, 0).

x y



$\int 1\,dy = \int \frac{1}{x}\,dx$

$y = \ln x + C$

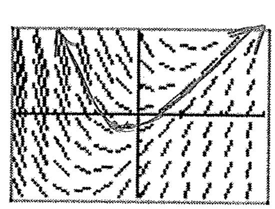
$0 = \ln 2 + C$

$-\ln 2 = C$

$y = \ln x - \ln 2$

b) Given $\frac{dy}{dx} = 2x - y$

Sketch the solution curve through the point $(-1, -1)$.



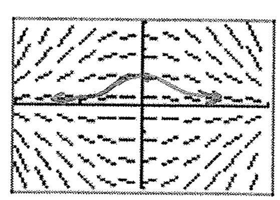
c) Given $\frac{dy}{dx} = -xy$

Sketch the solution curve through the point (0,2).

$\ln 2 = 0 + C$

$\ln y = \frac{-x^2}{2} + \ln 2$

$e^{\frac{-x^2}{2} + \ln 2} = y$



$\int \frac{1}{y}\,dy = \int -x\,dx$

$\ln y = \frac{-x^2}{2} + C$

Find the particular solution $y = f(x)$

4) $y(x^3+1)y' = x^2 \quad y(0) = -4$

$y' = \dfrac{x^2}{y(x^3+1)} \Rightarrow \dfrac{dy}{dx} = \dfrac{x^2}{y(x^3+1)}$

$\int y\,dy = \int \dfrac{x^2}{x^3+1}dx \qquad u = x^3+1, \; du = 3x^2 dx$

$\dfrac{y^2}{2} \quad \dfrac{1}{3}\int \dfrac{1}{u}du$

$\dfrac{y^2}{2} = \dfrac{1}{3}\ln(x^3+1) + C$

$\dfrac{(-4)^2}{2} = \dfrac{1}{3}(\ln(0^3+1)) + C$

$\dfrac{16}{2} = \dfrac{1}{3}\cdot 0 + C$

$8 = C$

$\dfrac{y^2}{2} = \dfrac{1}{3}\ln(x^3+1) + 8$

$y^2 = 2(\ln(x^3+1)^{1/3} + 8)$

$y = \sqrt{2(\ln(x^3+1)^{1/3} + 8)}$

Find the particular solution of the differential equation that satisfies the initial condition.

5a) $\dfrac{dy}{dt} = ky \qquad y(0) = 1000$

$\int \frac{1}{y}dy = \int k\,dt$

$\ln y = kt + C$

$\ln(1000) = k\cdot 0 + C$

$\ln y = kt + \ln 1000$

$e^{kt + \ln 1000} = y$

b) If $k = 0.13$ find $y(22)$.

$e^{(.13(22) + \ln 1000)} = y$

$\boxed{17,461.53 = y}$

6) Bacteria, which has a double-life of 12 days, grows from its initial amount to a population of 4539 in 40 days. What was the initial amount?

$y = y_0 e^{kt}$ # times doubling

$y = a\,b$ ↑ initial, growth factor

$4539 = a(2)^{40/12}$

$4539 = a(2)^{10/3}$

$450.32 = a$

$\approx \boxed{450.33}$

7) The half-life of kryptonite is 1200 years. Our ancestors buried the kryptonite many years ago and when we dig up the kryptonite there is 10% as much as when it was buried. How old is the kryptonite?

First

$\frac{1}{2} = 1e^{k\cdot 1200}$

$\ln\frac{1}{2} = 1200k$

$\dfrac{\ln 1/2}{1200} = k$

$y = ae^{kt}$

$10 = 100e^{(\frac{\ln 1/2}{1200}\cdot t)}$

$.1 = e^{(\frac{\ln 1/2}{1200}\cdot t)}$

$\ln .1 = \dfrac{\ln 1/2}{1200}\cdot t$

$t = \dfrac{\ln .1}{\ln 1/2 / 1200} = \boxed{3,986.3\text{yrs}}$

8) If you are a lifer at Stinky Burger and made \$9.00/hr. in 2004 and get a raise to \$10.00/hr. in 2006 due to being named Captain Stinky (nobody embodies the essence of Stinky like you do),

a) What is the rate of increase? b) What will your pay be in 2017 at this rate of growth?

Skip

$y = a(1+r)^t$

$10 = 9(1+r)^2$

$\frac{10}{9} = (1+r)^2$

$\sqrt{\frac{10}{9}} = 1+r$

$\sqrt{10/9} - 1 = r$

$r = .054$ or 5.4%

$y = 9(1+.054)^{13}$

$y = \$17.83$