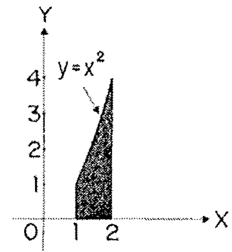
Chapter 5 – AP Calc MC Questions (Intro to Integration)

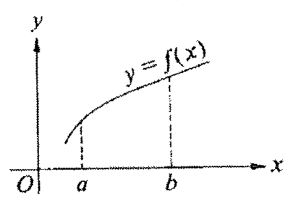
RAM & TRAPEZOIDAL RULE



D

42. Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x = \frac{4}{3}$ and $x = \frac{5}{3}$.

(A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$



27. If f is the continuous, strictly increasing function on the interval $a \le x \le b$ as shown above, which of the following must be true?

E

I. $\int_a^b f(x)\,dx < f(b)(b-a)$

II. $\int_a^b f(x)\,dx > f(a)(b-a)$

III. $\int_a^b f(x)\,dx = f(c)(b-a)$ for some number c such that $a < c < b$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

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x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

89. A table of values for a continuous function f is shown above. If four equal subintervals of $[0, 2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

B

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

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x	2	5	7	8
$f(x)$	10	30	40	20

85. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table above. Using the subintervals $[2,5]$, $[5,7]$, and $[7,8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$? C

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

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t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

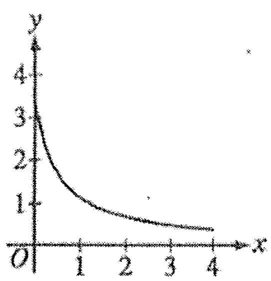
91. The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is E

(A) 26 ft/sec (B) 30 ft/sec (C) 37 ft/sec (D) 39 ft/sec (E) 41 ft/sec

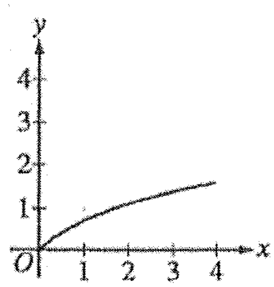
---

85. If a trapezoidal sum overapproximates $\int_0^4 f(x)\,dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

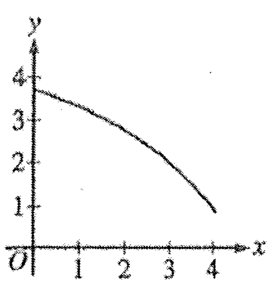
(A)



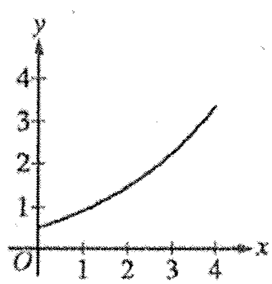
(B)



(C)

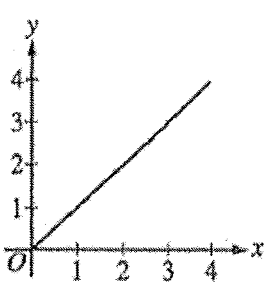


(D)



A

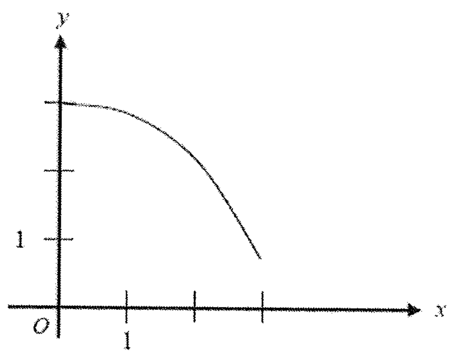
(E)



36. If the definite integral $\int_0^2 e^{x^2}\,dx$ is first approximated by using two inscribed rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is

D

(A) 53.60 (B) 30.51 (C) 27.80 (D) 26.80 (E) 12.78



Graph of f

10. The graph of function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

C

(A) $\int_1^3 f(x)\,dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(C) Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(E) Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

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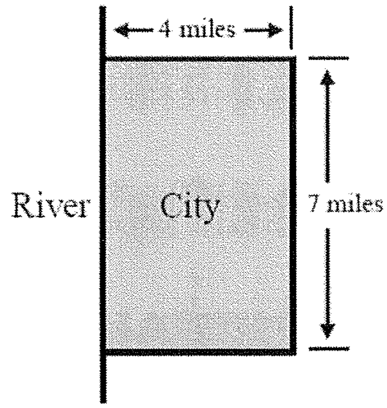
18. If three equal subdivisions of $[-4, 2]$ are used, what is the trapezoidal approximation of $\int_{-4}^{2} \frac{e^{-x}}{2}\,dx$?

E

(A) $e^2 + e^0 + e^{-2}$ (B) $e^4 + e^2 + e^0$ (C) $e^4 + 2e^2 + 2e^0 + e^{-2}$

(D) $\frac{1}{2}\left(e^4 + e^2 + e^0 + e^{-2}\right)$ (E) $\frac{1}{2}\left(e^4 + 2e^2 + 2e^0 + e^{-2}\right)$



92. A city located beside a river has a rectangular boundary as shown in the figure above. The population density of the city at any point along a strip x miles from the river's edge is $f(x)$ persons per square mile. Which of the following expressions gives the population of the city?

B

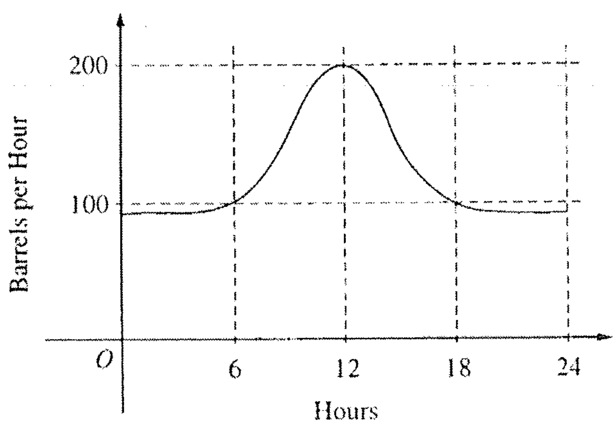
(A) $\int_0^4 f(x)\,dx$

(B) $7\int_0^4 f(x)\,dx$

(C) $28\int_0^4 f(x)\,dx$

(D) $\int_0^7 f(x)\,dx$

(E) $4\int_0^7 f(x)\,dx$



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

D

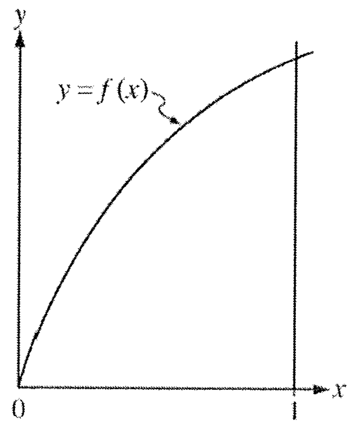
(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

x	0	2	4	6
$f(x)$	4	k	8	12

8. The function f is continuous on the closed interval $[0, 6]$ and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)\,dx$ found with 3 subintervals of equal length is 52. What is the value of k ?

(A) 2 (B) 6 (C) 7 (D) 10 (E) 14

D



80. A left Riemann sum, a right Riemann sum, and a trapezoidal sum are used to approximate the value of $\int_0^1 f(x)\,dx$, each using the same number of subintervals. The graph of the function f is shown in the figure above. Which of the sums give an underestimate of the value of $\int_0^1 f(x)\,dx$?

I. Left sum
II. Right sum
III. Trapezoidal sum

(A) I only
(B) II only
(C) III only
(D) I and III only

D

BASIC ANTIDERIVATIVES

1. $\int \left(x^3 - 3x\right) dx =$

(A) $3x^2 - 3 + C$ (B) $4x^4 - 6x^2 + C$ (C) $\frac{x^4}{3} - 3x^2 + C$

(D) $\frac{x^4}{4} - 3x + C$ (E) $\frac{x^4}{4} - \frac{3x^2}{2} + C$

E

---

2. $\int \frac{1}{x^2} dx =$

D

(A) $\ln x^2 + C$ (B) $-\ln x^2 + C$ (C) $x^{-1} + C$ (D) $-x^{-1} + C$ (E) $-2x^{-3} + C$

---

5. $\int_{-1}^{2} \frac{|x|}{x} dx$ is

B

(A) -3 (B) 1 (C) 2 (D) 3 (E) nonexistent

---

5. $\int \sec^2 x \, dx =$

(A) $\tan x + C$ (B) $\csc^2 x + C$ (C) $\cos^2 x + C$

A

(D) $\frac{\sec^3 x}{3} + C$ (E) $2\sec^2 x \tan x + C$

---

28. $\int_1^4 |x - 3| \, dx =$

C

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$ (E) 5

---

27. $\int_0^3 |x - 1| \, dx =$

D

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 6

---

31. $\int_0^2 \sqrt{4-x^2}\,dx =$

C

(A) $\frac{8}{3}$ (B) $\frac{16}{3}$ (C) π (D) 2π (E) 4π

---

33. Which of the following is equal to $\int_0^{\pi} \sin x\,dx$?

A

(A) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx$ (B) $\int_0^{\pi} \cos x\,dx$ (C) $\int_{-\pi}^{0} \sin x\,dx$

(D) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x\,dx$ (E) $\int_{\pi}^{2\pi} \sin x\,dx$

---

1. $\int_1^2 (4x^3 - 6x)\,dx =$

C

(A) 2
(B) 4
(C) 6
(D) 36
(E) 42

---

5. $\int_0^{\frac{\pi}{4}} \sin x\,dx =$

D

(A) $-\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $-\frac{\sqrt{2}}{2} - 1$ (D) $-\frac{\sqrt{2}}{2} + 1$ (E) $\frac{\sqrt{2}}{2} - 1$

---

5. $\int_0^x \sin t\,dt =$

E

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

---

20. What are all values of k for which $\int_{-3}^{k} x^2\,dx = 0$?

A

(A) -3 (B) 0 (C) 3 (D) -3 and 3 (E) -3, 0, and 3

---

38. If the second derivative of f is given by $f''(x) = 2x - \cos x$, which of the following could be $f(x)$?

(A) $\frac{x^3}{3} + \cos x - x + 1$

(B) $\frac{x^3}{3} - \cos x - x + 1$

(C) $x^3 + \cos x - x + 1$

(D) $x^2 - \sin x + 1$

(E) $x^2 + \sin x + 1$

A

---

27. If $\frac{dy}{dx} = \tan x$, then $y =$

(A) $\frac{1}{2}\tan^2 x + C$ (B) $\sec^2 x + C$ (C) $\ln|\sec x| + C$

(D) $\ln|\cos x| + C$ (E) $\sec x \tan x + C$

C

---

12. If $f'(x) = \frac{2}{x}$ and $f(\sqrt{e}) = 5$, then $f(e) =$

(A) 2 (B) ln 25 (C) $5 + \frac{2}{e} - \frac{2}{e^2}$ (D) 6 (E) 25

D

PROPERTIES OF DEFINITE INTEGRALS

38. If $\int_1^2 f(x-c)\,dx = 5$ where c is a constant, then $\int_{1-c}^{2-c} f(x)\,dx =$

(A) $5+c$ (B) 5 (C) $5-c$ (D) $c-5$ (E) -5

B

---

39. If $\int_1^{10} f(x)\,dx = 4$ and $\int_{10}^{3} f(x)\,dx = 7$, then $\int_1^3 f(x)\,dx =$

(A) -3 (B) 0 (C) 3 (D) 10 (E) 11

E

---

12. If f and g are continuous functions, and if $f(x) \geq 0$ for all real numbers x, which of the following must be true?

I. $\int_a^b f(x)g(x)dx = \left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right)$

II. $\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

III. $\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)dx}$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

B

---

26. If f is a function such that $f'(x)$ exists for all x and $f(x) > 0$ for all x, which of the following is NOT necessarily true?

(A) $\int_{-1}^{1} f(x)\,dx > 0$

(B) $\int_{-1}^{1} 2f(x)\,dx = 2\int_{-1}^{1} f(x)\,dx$

(C) $\int_{-1}^{1} f(x)\,dx = 2\int_{0}^{1} f(x)\,dx$

(D) $\int_{-1}^{1} f(x)\,dx = -\int_{1}^{-1} f(x)\,dx$

(E) $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$

C

---

42. If $\int_1^4 f(x)\,dx = 6$, what is the value of $\int_1^4 f(5-x)\,dx$?

A

(A) 6 (B) 3 (C) 0 (D) −1 (E) −6

---

9. If $\int_{-1}^{1} e^{-x^2}\,dx = k$, then $\int_{-1}^{0} e^{-x^2}\,dx =$

D

(A) $-2k$ (B) $-k$ (C) $-\frac{k}{2}$ (D) $\frac{k}{2}$ (E) $2k$

---

32. If $\int_a^b f(x)dx = 5$ and $\int_a^b g(x)dx = -1$, which of the following must be true?

I. $f(x) > g(x)$ for $a \le x \le b$

II. $\int_a^b (f(x)+g(x))\,dx = 4$

III. $\int_a^b (f(x)g(x))\,dx = -5$

B

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

---

40. Let f be a continuous function on the closed interval $[0,2]$. If $2 \le f(x) \le 4$, then the greatest possible value of $\int_0^2 f(x)\,dx$ is

D

(A) 0 (B) 2 (C) 4 (D) 8 (E) 16

---

3. If $\int_a^b f(x)\,dx = a+2b$, then $\int_a^b (f(x)+5)\,dx =$

C

(A) $a+2b+5$ (B) $5b-5a$ (C) $7b-4a$ (D) $7b-5a$ (E) $7b-6a$

---

79. If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_5^2 f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

B

(A) −21 (B) −13 (C) 0 (D) 13 (E) 21

---

82. If $f(x) = g(x) + 7$ for $3 \le x \le 5$, then $\int_3^5 [f(x) + g(x)]\,dx =$

(A) $2\int_3^5 g(x)\,dx + 7$

(B) $2\int_3^5 g(x)\,dx + 14$

(C) $2\int_3^5 g(x)\,dx + 28$

(D) $\int_3^5 g(x)\,dx + 7$

(E) $\int_3^5 g(x)\,dx + 14$

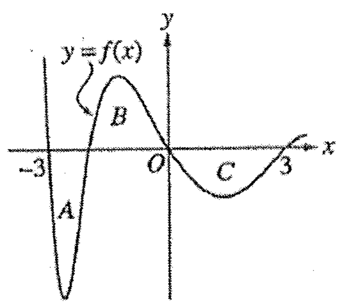
B

---

11. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

A



C

77. The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3} (f(x) + 1)\,dx$?

(A) −2 (B) −1 (C) 4 (D) 7 (E) 12

---

77. If $\int_0^3 f(x)\,dx = 6$ and $\int_3^5 f(x)\,dx = 4$, then $\int_0^5 (3 + 2f(x))\,dx =$

(A) 10 (B) 20 (C) 23 (D) 35 (E) 50

D

BASIC AVERAGE VALUE PROBLEMS

33. What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \le t \le 2$?

A

(A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

---

21. The average value of $\frac{1}{x}$ on the closed interval $[1,3]$ is

D

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{\ln 2}{2}$ (D) $\frac{\ln 3}{2}$ (E) $\ln 3$

---

36. What is the average value of y for the part of the curve $y = 3x - x^2$ which is in the first quadrant?

C

(A) -6 (B) -2 (C) $\frac{3}{2}$ (D) $\frac{9}{4}$ (E) $\frac{9}{2}$

---

85. Let f be a twice differentiable function such that $f(1) = 2$ and $f(3) = 7$. Which of the following must be true for the function f on the interval $1 \le x \le 3$?

I. The average rate of change of f is $\frac{5}{2}$.

II. The average value of f is $\frac{9}{2}$.

III. The average value of f' is $\frac{5}{2}$.

D

(A) None
(B) I only
(C) III only
(D) I and III only
(E) II and III only

---

44. If f is continuous on the interval $[a,b]$, then there exists c such that $a < c < b$ and $\int_a^b f(x)dx =$

E

(A) $\frac{f(c)}{b-a}$ (B) $\frac{f(b)-f(a)}{b-a}$ (C) $f(b)-f(a)$ (D) $f'(c)(b-a)$ (E) $f(c)(b-a)$

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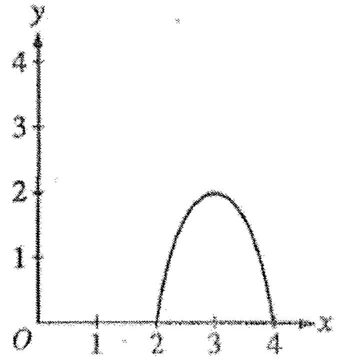
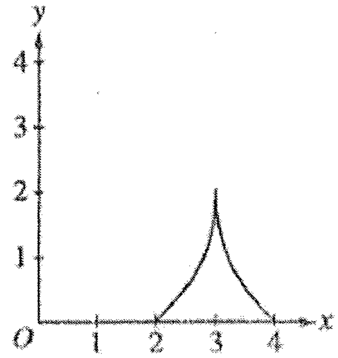
88. On the closed interval [2, 4], which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$?

(A)


(B)


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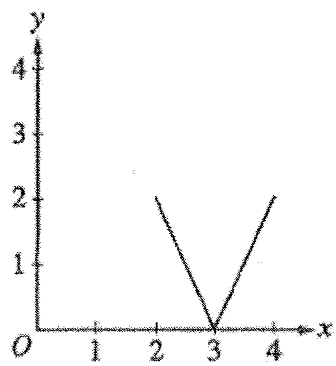

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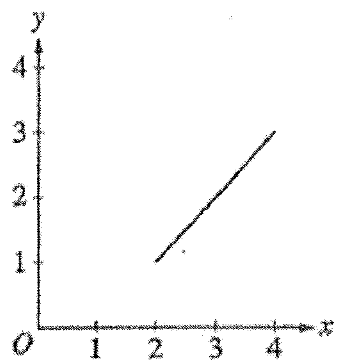
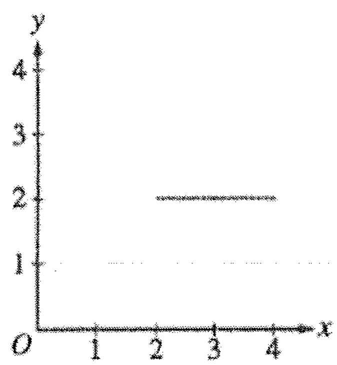


(E)


C

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34. The average value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

C

(A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

FTC PART I

12. If $F(x)=\int_0^x e^{-t^2}\,dt$, then $F'(x)=$

(A) $2xe^{-x^2}$ (B) $-2xe^{-x^2}$ (C) $\dfrac{e^{-x^2+1}}{-x^2+1}-e$

(D) $e^{-x^2}-1$ (E) e^{-x^2}

E

---

42. $\dfrac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt=$

(A) $\dfrac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}-5$ (C) $\sqrt{1+x^2}$

(D) $\dfrac{x}{\sqrt{1+x^2}}-\dfrac{1}{\sqrt{5}}$ (E) $\dfrac{1}{2\sqrt{1+x^2}}-\dfrac{1}{2\sqrt{5}}$

C

---

25. For all $x>1$, if $f(x)=\int_1^x \frac{1}{t}\,dt$, then $f'(x)=$

(A) 1 (B) $\dfrac{1}{x}$ (C) $\ln x-1$ (D) $\ln x$ (E) e^x

B

---

14. If $F(x)=\int_1^{x^2} \sqrt{1+t^3}\,dt$, then $F'(x)=$

(A) $2x\sqrt{1+x^6}$ (B) $2x\sqrt{1+x^3}$ (C) $\sqrt{1+x^6}$

(D) $\sqrt{1+x^3}$ (E) $\int_1^{x^2} \dfrac{3t^2}{2\sqrt{1+t^3}}\,dt$

A

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41. $\dfrac{d}{dx}\int_0^x \cos(2\pi u)\,du$ is

(A) 0 (B) $\dfrac{1}{2\pi}\sin x$ (C) $\dfrac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

D

---

15. If $F(x)=\int_0^x \sqrt{t^3+1}\,dt$, then $F'(2)=$

(A) -3 (B) -2 (C) 2 (D) 3 (E) 18

D

---

27. If f is the function given by $f(x) = \int_4^{2x} \sqrt{t^2 - t}\, dt$, then $f'(2) =$

E

(A) 0 (B) $\frac{7}{2\sqrt{12}}$ (C) $\sqrt{2}$ (D) $\sqrt{12}$ (E) $2\sqrt{12}$

---

23. $\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\, dt\right) =$

E

(A) $-\cos(x^6)$ (B) $\sin(x^3)$ (C) $\sin(x^6)$ (D) $2x\sin(x^3)$ (E) $2x\sin(x^6)$

---

88. Let $f(x) = \int_0^{x^2} \sin t\, dt$. At how many points in the closed interval $\left[0, \sqrt{\pi}\right]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

C

(A) Zero
(B) One
(C) Two
(D) Three
(E) Four

---

41. Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2}\, dt$. At what value of x is $f(x)$ a minimum?

C

(A) For no value of x (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

---

92. Let g be the function given by $g(x) = \int_0^x \sin(t^2)dt$ for $-1 \le x \le 3$. On which of the following intervals is g decreasing?

(A) $-1 \le x \le 0$
(B) $0 \le x \le 1.772$
(C) $1.253 \le x \le 2.171$
(D) $1.772 \le x \le 2.507$
(E) $2.802 \le x \le 3$

D

---

22. If $f(x)=\int_0^x \frac{1}{\sqrt{t^3+2}}\,dt$, which of the following is FALSE?

(A) $f(0)=0$

(B) f is continuous at x for all $x \geq 0$.

(C) $f(1)>0$

(D) $f'(1)=\frac{1}{\sqrt{3}}$

(E) $f(-1)>0$

E

FTC PART II

20. If F and f are continuous functions such that $F'(x) = f(x)$ for all x, then $\int_a^b f(x)\,dx$ is

(A) $F'(a) - F'(b)$
(B) $F'(b) - F'(a)$
(C) $F(a) - F(b)$
(D) $F(b) - F(a)$
(E) none of the above

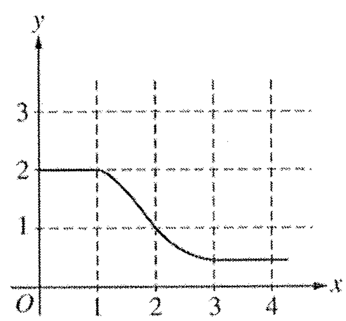
D

---

13. If the function f has a continuous derivative on $[0, c]$, then $\int_0^c f'(x)\,dx =$

A

(A) $f(c) - f(0)$ (B) $|f(c) - f(0)|$ (C) $f(c)$ (D) $f(x) + c$ (E) $f''(c) - f''(0)$



D

78. The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

---

82. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)\,dx =$

(A) $2F(3) - 2F(1)$
(B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$
(C) $2F(6) - 2F(2)$
(D) $F(6) - F(2)$
(E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

E

---

x	-4	-3	-2	-1
$f(x)$	0.75	-1.5	-2.25	-1.5
$f'(x)$	-3	-1.5	0	1.5

85. The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval $[-4, -1]$, what is the value of $\int_{-4}^{-1} f'(x)\,dx$?

B

(A) -4.5 (B) -2.25 (C) 0 (D) 2.25 (E) 4.5

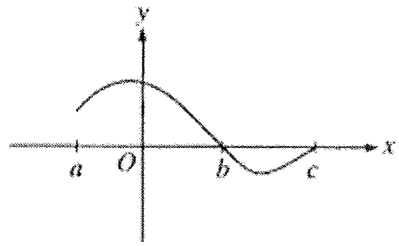
---

20. If $g(x) = x^2 - 3x + 4$ and $f(x) = g'(x)$, then $\int_1^3 f(x)\,dx =$

C

(A) $-\frac{14}{3}$ (B) -2 (C) 2 (D) 4 (E) $\frac{14}{3}$

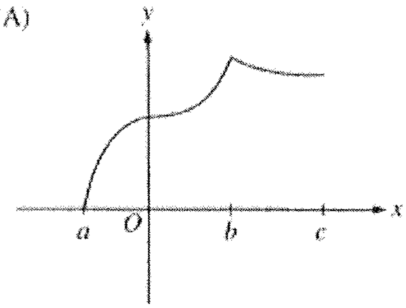
READING VELOCITY GRAPHS



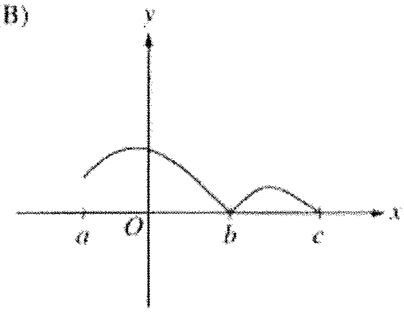
88. Let $f(x) = \int_a^x h(t)\,dt$, where h has the graph shown above. Which of the following could be the graph of f?

E

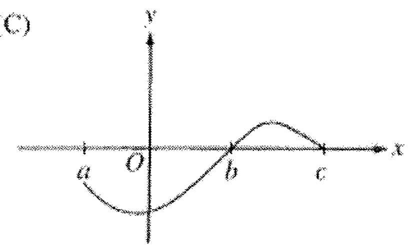
(A)



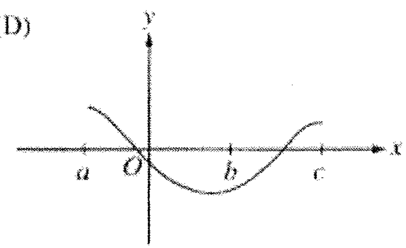
(B)



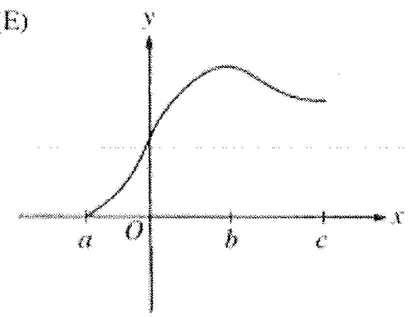
(C)



(D)

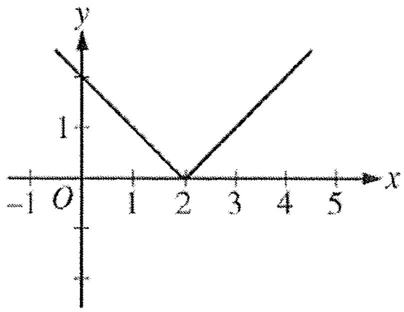


(E)

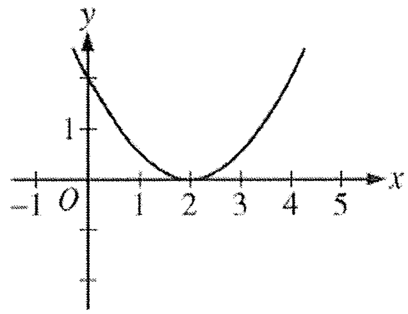


16. If $f'(x) = |x - 2|$, which of the following could be the graph of $y = f(x)$?

(A)

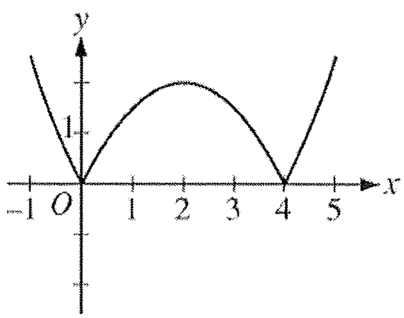


(B)

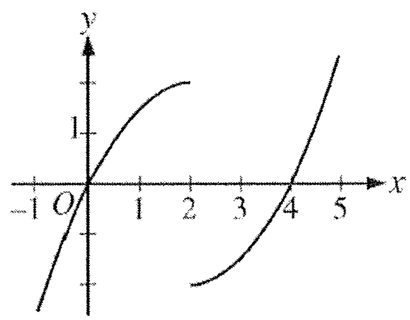


E

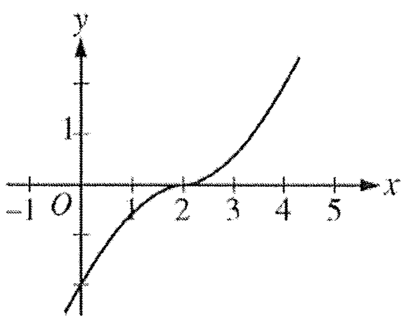
(C)

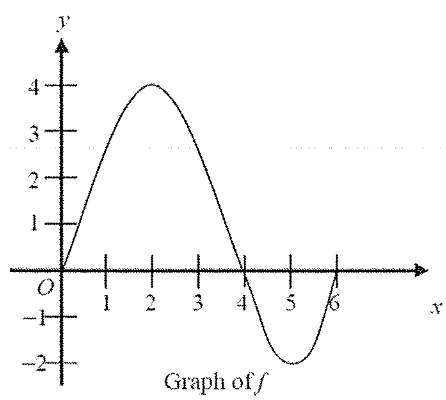


(D)



(E)





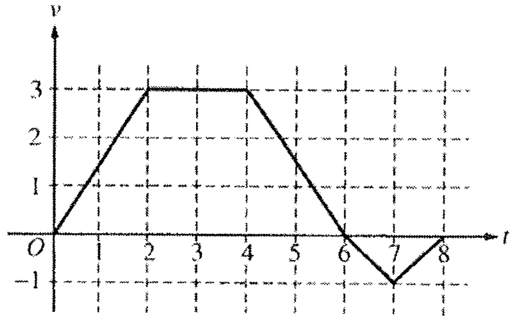
Graph of f

C

17. The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

Questions 8-9 refer to the following situation.



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

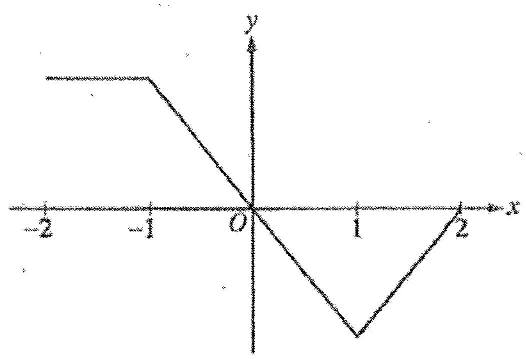
8. At what value of t does the bug change direction? C

(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

---

9. What is the total distance the bug traveled from $t = 0$ to $t = 8$? B

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6



Graph of f'

7. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f ? B

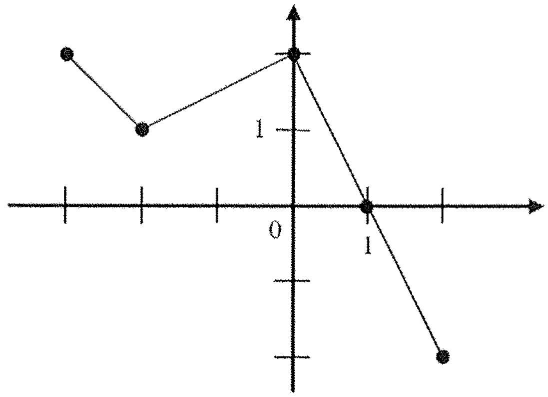
(A) f is decreasing for $-1 \le x \le 1$.

(B) f is increasing for $-2 \le x \le 0$.

(C) f is increasing for $1 \le x \le 2$.

(D) f has a local minimum at $x = 0$.

(E) f is not differentiable at $x = -1$ and $x = 1$.

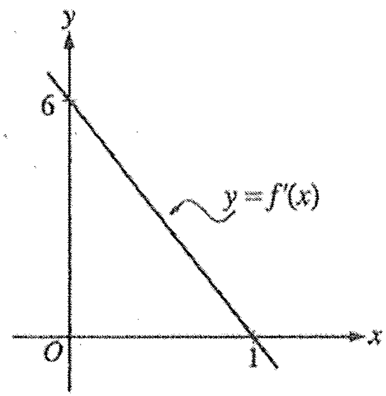


Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x)=\int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

D

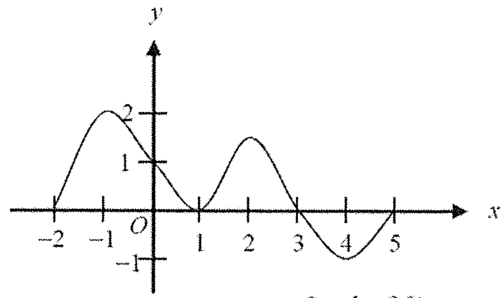
(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$



D

22. The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0 (B) 3 (C) 6 (D) 8 (E) 11



Graph of f'

76. The graph of f', the derivative f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

B

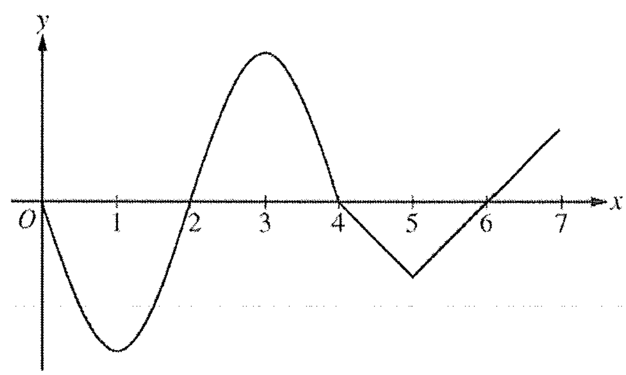
(A) [−2, 1] only

(B) [−2, 3]

(C) [3, 5] only

(D) [0, 1.5] and [3, 5]

(E) [−2, −1], [1, 2], and [4, 5]



Graph of f'

E

84. The graph of f', the derivative of the function f, is shown above. On which of the following intervals is f decreasing?

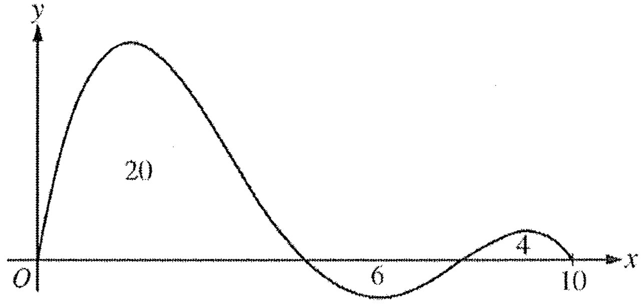
(A) [2, 4] only

(B) [3, 5] only

(C) [0, 1] and [3, 5]

(D) [2, 4] and [6, 7]

(E) [0, 2] and [4, 6]

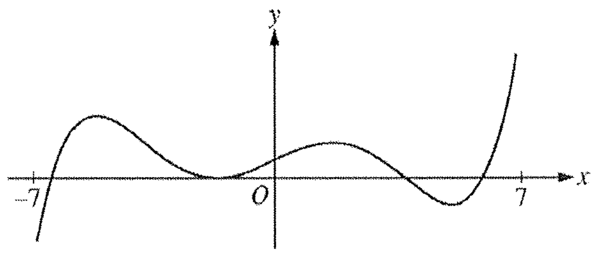


Graph of f'

C

21. The graph of f', the derivative of the function f, is shown above for $0 \le x \le 10$. The areas of the regions between the graph of f' and the x-axis are 20, 6, and 4, respectively. If $f(0) = 2$, what is the maximum value of f on the closed interval $0 \le x \le 10$?

(A) 16 (B) 20 (C) 22 (D) 30 (E) 32



Graph of f'

A

87. The figure above shows the graph of f', the derivative of the function f, on the open interval $-7 < x < 7$. If f' has four zeros on $-7 < x < 7$, how many relative maxima does f have on $-7 < x < 7$?

(A) One (B) Two (C) Three (D) Four (E) Five

PARTICLE MOTION APPLICATION

13. The acceleration α of a body moving in a straight line is given in terms of time t by $\alpha = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t, what is $s(4) - s(2)$?

D

(A) 20 (B) 24 (C) 28 (D) 32 (E) 42

---

28. A point moves in a straight line so that its distance at time t from a fixed point of the line is $8t - 3t^2$. What is the *total* distance covered by the point between $t = 1$ and $t = 2$?

C

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) 5

---

87. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is -2. For what value t will the velocity of the particle be zero?

B

(A) 1.02 (B) 1.48 (C) 1.85 (D) 2.81 (E) 3.14

---

7. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at $t = 1$?

B

(A) 4 (B) 6 (C) 9 (D) 11 (E) 12

---

9. A particle moves along the x-axis so that at any time $t > 0$, its velocity is given by $v(t) = 4 - 6t^2$. If the particle is at position $x = 7$ at time $t = 1$, what is the position of the particle at time $t = 2$?

C

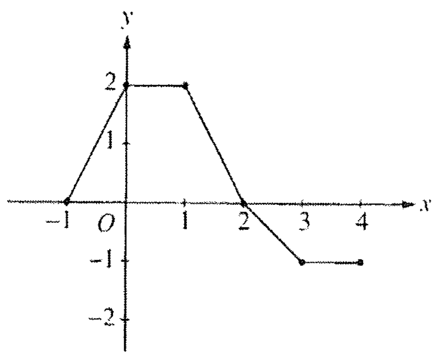
(A) -10 (B) -5 (C) -3 (D) 3 (E) 17

MISCELLANEOUS

41. If $\begin{cases} f(x) = 8 - x^2 & \text{for } -2 \le x \le 2, \\ f(x) = x^2 & \text{elsewhere}, \end{cases}$ then $\int_{-1}^{3} f(x)\,dx$ is a number between

D

(A) 0 and 8 (B) 8 and 16 (C) 16 and 24 (D) 24 and 32 (E) 32 and 40



2. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

B

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

---

4. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

(B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \le x \le b$.

(D) f has a maximum value on $a \le x \le b$.

(E) $\int_a^b f(x)dx$ exists.

B

---

38. Let f and g have continuous first and second derivatives everywhere. If $f(x) \le g(x)$ for all real x, which of the following must be true?

I. $f'(x) \le g'(x)$ for all real x
II. $f''(x) \le g''(x)$ for all real x
III. $\int_0^1 f(x)\,dx \le \int_0^1 g(x)\,dx$

C

(A) None (B) I only (C) III only (D) I and II only (E) I, II, and III
