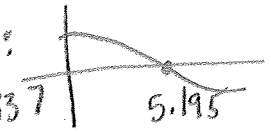
Chapter 7 Review
Honors Calculus

Name Key Hour

1. For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

$v(t)$ graph: 

$v(5.5) = -.45337$

$a(5.5) = -1.3581$

At $t = 5.5$

$v(t) < 0$, $A(t) < 0$

Speed is increasing

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

$\int_0^6 (2\sin(e^{t/4})+1)\,dt$

$\frac{\text{displacement}}{\text{time}} = \frac{f(b)-f(a)}{b-a} = \frac{11.6928}{6-0} = 1.94938$

$\int_0^6 (2\sin(e^{t/4})+1)\,dt = 11.69628$ or $\frac{1}{6}\int_0^6 v(t) = 1.949$

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

$\int_0^6 \left|(2\sin e^{t/4}+1)\right| dt = 12.573083$

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

$v(t) = 0$

$t = 5.195$

$x(0) = 2$

$v(t)$ switches from (+) to (−)

$\int_0^{5.195} (2\sin e^{t/4}+1)\,dt = 12.134$

$2 + 12.134 = 14.134$

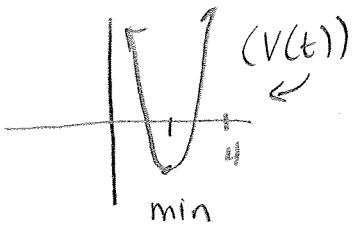
2. **The maximum velocity attained on the interval $0 \le t \le 5$ by the particle whose displacement is given by $s(t) = 2t^3 - 12t^2 + 16t + 2$ is**

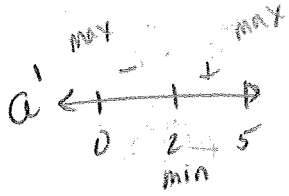
(A) 286 **(B) 46** **(C) 16** **(D) 0** **(E) −8**

$v(t) = 6t^2 - 24t + 16 =$

$a(t) = 12t - 24 = 0$ $12t = 24$, $t = 2$

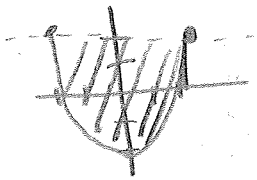
$v(0) = 16$ $v(5) = 46$





Find the area of the region between the two curves in each problem, and be sure to sketch each one.

3. The curve $y = x^2 - 2$ and the line $y = 2$. $\boxed{\frac{32}{3}}$



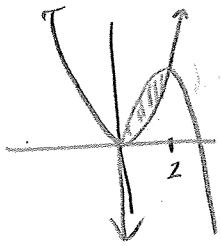
$x^2 - 2 = 2$

$x^2 = 4$

$x = \pm 2$

$\int_{-2}^{2} (2 - (x^2 - 2))\,dx = \int_{-2}^{2} (-x^2 + 4)\,dx = \boxed{10.66 \text{ or } \frac{32}{3}}$

4. The curve $y = x^2$ and the curve $y = 4x - x^2$. $\boxed{\frac{8}{3}}$



$4x - x^2 = x^2$

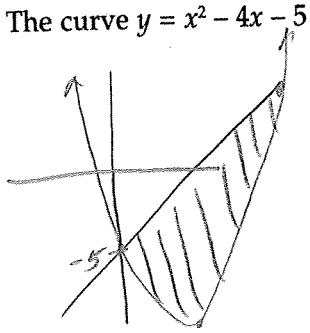
$2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0, x = 2$

$\int_{0}^{2} (4x - x^2 - x^2)\,dx = \boxed{2.6 \text{ or } \frac{8}{3}}$

5. The curve $y = x^2 - 4x - 5$ and the curve $y = 2x - 5$. $\boxed{36}$



$x^2 - 4x - 5 = 2x - 5$

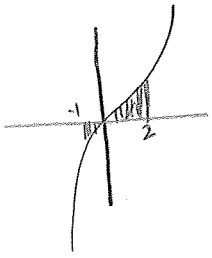
$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0, x = 6$

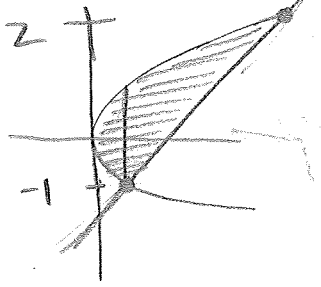
$\int_{0}^{6} (2x - 5 - (x^2 - 4x - 5)) = \int_{0}^{6} (-x^2 + 6x) = \boxed{36}$

6. The curve $y = x^3$ and the x-axis, from $x = -1$ to $x = 2$. $\boxed{\frac{17}{4}}$



$\int_{-1}^{2} |x^3|\,dx = \boxed{4.25}$

7. The curve $x = y^2$ and the line $x = y + 2$. $\boxed{\frac{9}{2}}$



$\int_{-1}^{2} ((y+2) - y^2)\,dy = \boxed{4.5}$

$y + 2 = y^2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

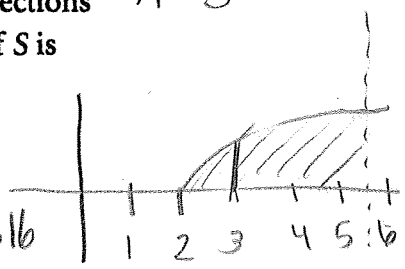
$y = 2 \quad y = -1$

8. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln(x-1)}$, the line $x = 2e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

$A = s^2$

(A) 1.587. (B) 2.173. (C) 3.185.

(D) 3.501. (E) 6.347.



$\ln(x-1) > 0$

$x-1 > e \quad x > 2$

$\int_2^{5.43} \left(\sqrt{\ln(x-1)}\right)^2 dx = 3.16$

9. A region in the first quadrant is enclosed by the graphs of $y = e^{2x}$, $x = 1$, and the coordinate axes. If the region is rotated about the y-axis, the volume of the solid that is generated is represented by which of the following integrals?

Washer

$\pi \int_0^{e^2} 1^2 - \pi \int_1^{e^2} \left(\frac{\ln y}{2}\right)^2$

(A) $2\pi \int_0^1 xe^{2x}\,dx$ (C) $\pi \int_0^1 e^{4x}\,dx$ (E) $\frac{\pi}{4} \int_0^e \ln^2 y\,dy$

(B) $2\pi \int_0^1 e^{2x}\,dx$ (D) $\pi \int_0^e y \ln y\,dy$

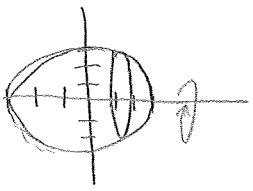
or

Shell

$2\pi \int_0^1 xe^{2x}\,dx$

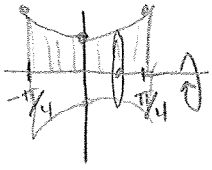
Calculate the volumes below.

10. Find the volume of the solid that results when the region bounded by $y = \sqrt{9-x^2}$ and the x-axis is revolved around the x-axis. 36π



$\pi \int_{-3}^{3} \left(\sqrt{9-x^2}\right)^2 dx = 36\pi \approx 113.097$

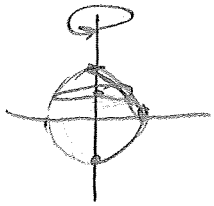
11. Find the volume of the solid that results when the region bounded by $y = \sec x$ and the x-axis from $x = -\frac{\pi}{4}$ to $x = \frac{\pi}{4}$ is revolved around the x-axis. 2π



$\pi \int_{-\pi/4}^{\pi/4} \sec^2 x\,dx = 2\pi \approx 6.28$

12. Find the volume of the solid that results when the region bounded by $x = 1 - y^2$ and the y-axis is revolved around the y-axis.

$\frac{16\pi}{15}$



$y^2 = 1 - x$

$y = \sqrt{1-x}$

$\pi \int_{-1}^{1} (1-y^2)^2\,dy = 1.06\pi =$

OR Shells

$2\pi r h$

$r = x$

$h = \sqrt{1-x} - -\sqrt{1-x}$

$\int_0^1 2\pi x (2\sqrt{1-x})\,dx$