Differentiation - Power, Constant, and Sum Rules

Date________________ Period____

Differentiate each function with respect to *x*.

1) $y = 5$

$\frac{dy}{dx} = \boxed{0}$

2) $f(x) = 5x^{18}$

$f'(x) = 5 \cdot 18x^{17} = \boxed{90x^{17}}$

3) $y = 4x^5 + x$

$\frac{dy}{dx} = \boxed{20x^4 + 1}$

4) $f(x) = 4x^4 - 5x - 3$

$f'(x) = 16x^3 - 5$

5) $y = 3x^{\frac{5}{4} - \frac{4}{4}}$

$\frac{dy}{dx} = 3 \cdot \frac{5}{4} x^{1/4} = \boxed{\frac{15}{4} x^{1/4}}$

6) $y = \frac{5}{4} x^{\frac{2}{3}}$

$\frac{dy}{dx} = \frac{5}{4} \cdot \frac{2}{3} x^{-1/3} = \frac{10}{12x^{1/3}} = \frac{5}{6x^{1/3}}$

7) $y = -4x^{-5}$

$\frac{dy}{dx} = 20x^{-6} = \boxed{\frac{20}{x^6}}$

8) $y = \frac{3}{x^3} = 3x^{-3}$

$\frac{dy}{dx} = -9x^{-4} = \boxed{\frac{-9}{x^4}}$

9) $y = x^{\frac{2}{3}}$

$\frac{dy}{dx} = \frac{2}{3} x^{-1/3} = \boxed{\frac{2}{3x^{1/3}}}$

10) $f(x) = -2\sqrt[4]{x} = -2x^{1/4}$

$f'(x) = -2 \cdot \frac{1}{4} x^{-3/4}$

$= -\frac{1}{2x^{3/4}}$

11) $y = \frac{2}{3}x^4 + 5x - x^{-3}$

$\frac{dy}{dx} = \frac{8}{3}x^3 + 5 + 3x^{-4}$

$\frac{dy}{dx} = \frac{8}{3}x^3 + 5 + \frac{3}{x^4}$

12) $y = -\frac{1}{2}x^4 + 3x^{\frac{5}{3}} + 2x$

$\frac{dy}{dx} = -2x^3 + 5x^{2/3} + 2$

Differentiate each function with respect to the given variable.

13) $y = -3r^5 - 5r^2$

$\frac{dr}{dy} = -15r^4 - 10r$

14) $f(s) = -\frac{3}{s^2} - \frac{4}{s^4} = -3s^{-2} - 4s^{-4}$

$f'(s) = 6s^{-3} + 16s^{-5}$

$f'(s) = \frac{6}{s^3} + \frac{16}{s^5}$

15) $f(x) = \frac{2}{3}x^{\frac{3}{2}} - \frac{3}{4}x^{\frac{3}{5}}$

$f'(x) = x^{1/2} - \frac{9}{20}x^{-2/5}$

$f'(x) = x^{1/2} - \frac{9}{20x^{2/5}}$

16) $h(s) = \sqrt{2} \cdot \sqrt[3]{s} + \sqrt{2} \cdot \sqrt[5]{s}$

$= \sqrt{2}s^{1/3} + \sqrt{2}s^{1/5}$

$= \frac{\sqrt{2}}{3}s^{-2/3} + \frac{\sqrt{2}}{5}s^{-4/5}$

$= \frac{\sqrt{2}}{3s^{2/3}} + \frac{\sqrt{2}}{5s^{4/5}}$

Differentiate each function with respect to x. Problems may contain constants a, b, and c.

17) $y = 5c$

$\frac{dy}{dx} = 0$

18) $y = 4ax^{3a} - bx^{3c}$

$\frac{dy}{dx} = 4a \cdot 3a\,x^{3a-1} - b \cdot 3c\,x^{3c-1}$

$\frac{dy}{dx} = 12a^2x^{3a-1} - 3bc\,x^{3c-1}$

Name________________________

Differentiation - Quotient Rule

Date________________ Period____

Differentiate each function with respect to x.

1) $y = \dfrac{2}{2x^4 - 5}$

$\dfrac{dy}{dx} = \dfrac{(2x^4-5)(0) - (2)(8x^3)}{(2x^4-5)^2}$

$= \dfrac{-16x^3}{(2x^4-5)^2}$

2) $f(x) = \dfrac{2}{x^5 - 5}$

$\dfrac{dy}{dx} = \dfrac{(x^5-5)\cdot 0 - (5x^4)(2)}{(x^5-5)^2}$

$\dfrac{dy}{dx} = \dfrac{-10x^4}{(x^5-5)^2}$

3) $f(x) = \dfrac{5}{4x^3 + 4}$

$f'(x) = \dfrac{(4x^3+4)(0) - (5)(12x^2)}{(4x^3+4)^2}$

$f'(x) = \dfrac{-60x^2}{(4x^3+4)^2}$

4) $y = \dfrac{4x^3 - 3x^2}{4x^5 - 4}$

$\dfrac{dy}{dx} = \dfrac{(4x^5-4)(12x^2-6x) - (20x^4)(4x^3-3x^2)}{(4x^5-4)^2}$

5) $y = \dfrac{3x^4 + 2}{3x^3 - 2}$

$\dfrac{dy}{dx} = \dfrac{(3x^3-2)(12x^3) - (3x^4+2)(9x^2)}{(3x^3-2)^2}$

6) $y = \dfrac{4x^5 + 2x^2}{3x^4 + 5}$

$\dfrac{dy}{dx} = \dfrac{(3x^4+5)(20x^4+4x) - (4x^5+2x^2)(12x^3)}{(3x^4+5)^2}$

7) $y = \dfrac{4x^5 + x^2 + 4}{5x^2 - 2}$

$\dfrac{dy}{dx} = \dfrac{(5x^2-2)(20x^4+2x) - (4x^5+x^2+4)(10x)}{(5x^2-2)^2}$

8) $y = \dfrac{3x^4 + 5x^3 - 5}{2x^4 - 4}$

$\dfrac{dy}{dx} = \dfrac{(2x^4-4)(12x^3+15x^2) - (3x^4+5x^3-5)(8x^3)}{(2x^4-4)^2}$

9) $y = \dfrac{x^3 - x^2 - 3}{x^5 + 3}$

$$\frac{dy}{dx} = \frac{(x^5+3)(3x^2-2x) - (x^3-x^2-3)(5x^4)}{(x^5+3)^2}$$

10) $y = \dfrac{x^4 + 6}{3 - 4x^{-4}}$

$$\frac{dy}{dx} = \frac{(3-4x^{-4})(4x^3) - (x^4+6)(16x^{-5})}{(3-4x^{-4})^2}$$

11) $y = \dfrac{4x^4 - 4x^2 + 5}{2x^{\frac{5}{3}} + 3}$

$$\frac{dy}{dx} = \frac{(2x^{5/3}+3)(16x^3-8x) - (4x^4-4x^2+5)(\frac{10}{3}x^{2/3})}{(2x^{5/3}+3)^2}$$

Critical thinking question:

12) A classmate claims that $\left(\dfrac{f}{g}\right)' = \dfrac{f'}{g'}$ for any functions f and g. Show an example that proves your classmate wrong.

$f = 4$

$g = 2x$

$$\left(\frac{f}{g}\right)' = \left(\frac{4}{2x}\right)' = \frac{2x \cdot 0 - 4(2)}{(2x)^2} = \frac{-8}{4x^2} = \frac{-2}{x^2}$$

$f' = 0$

$g' = 2$

$$\frac{f'}{g'} = \frac{0}{2} = 0$$

Not equal!

Kuta Software - Infinite Calculus

Name Key

Differentiation - Product Rule $vu' + uv'$

Date________________ Period____

Differentiate each function with respect to *x*.

1) $y = -x^3(3x^4 - 2)$ ($u = -x^3$, $v = 3x^4 - 2$)

$y' = (3x^4 - 2)(-3x^2) + (-x^3)(12x^3)$

2) $f(x) = x^2(-3x^2 - 2)$ ($u = x^2$, $v = -3x^2 - 2$)

$f'(x) = (-3x^2 - 2)(2x) + x^2(-6x)$

3) $y = (-2x^4 - 3)(-2x^2 + 1)$ (u, v)

$\frac{dy}{dx} = (-2x^2 + 1)(-8x^3) + (-2x^4 - 3)(-4x)$

4) $f(x) = (2x^4 - 3)(x^2 + 1)$ (u, v)

$f'(x) = (x^2 + 1)(8x^3) + (2x^4 - 3)(2x)$

5) $f(x) = (5x^5 + 5)(-2x^5 - 3)$ (u, v)

$f'(x) = (-2x^5 - 3)(25x^4) + (5x^5 + 5)(-10x^4)$

6) $f(x) = (-3 + x^{-3})(-4x^3 + 3)$ (u, v)

$f'(x) = (-4x^3 + 3)(-3x^{-4}) + (-3 + x^{-3})(-12x^2)$

7) $y = (-2x^4 + 5x^2 + 4)(-3x^2 + 2)$ (u, v)

$y' = (-3x^2 + 2)(-8x^3 + 5x) + (-2x^4 + 5x^2 + 4)(-6x)$

8) $y = (x^4 + 3)(-4x^5 + 5x^4 + 5)$ (u, v)

$y' = (-4x^5 + 5x^4 + 5)(4x^3) + (x^4 + 3)(-20x^4 + 20x^3)$

9) $y = \overset{u}{(5x^4 - 3x^2 - 1)}\overset{v}{(-5x^2 + 3)}$

$\frac{dy}{dx} = (-5x^2+3)(20x^3-6x) + (5x^4-3x^2-1)(-10x)$

10) $f(x) = \overset{u}{\left(-10x^2 - 7\sqrt[5]{x^2} + 9\right)}\overset{v}{(2x^3 + 4)}$

$f'(x) = (2x^3+4)\left(-20x - \frac{14}{5}x^{-3/5} + 9\right) + \left(-10x^2 - 7\sqrt[5]{x^2} + 9\right)(6x^2)$

11) $y = \overset{u}{(5 + 3x^{-2})}\overset{v}{(4x^5 + 6x^3 + 10)}$

$y' = (4x^5+6x^3+10)(-6x^{-3}) + (5+3x^{-2})(20x^4+18x^2)$

12) $y = \overset{u}{(-6x^4 + 2 + 6x^{-4})}\overset{v}{(6x^4 + 7)}$

$y' = (6x^4+7)(-24x^3-24x^{-5}) + (-6x^4+2+6x^{-4})(24x^3)$

13) $f(x) = \overset{u}{\left(-7x^4 + 10x^{\frac{2}{5}} + 8\right)}\overset{v}{(x^2 + 10)}$

$f'(x) = (x^2+10)(-28x^3 + 4x^{-3/5}) + (-7x^4 + 10x^{2/5} + 8)(2x)$

Critical thinking question:

14) A classmate claims that $(f \cdot g)' = f' \cdot g'$ for any functions f and g. Show an example that proves your classmate wrong.

$f = 2x \qquad (f \cdot g)' = (2x \cdot 4)' = (8x)' = 8$

$g = 4 \qquad f' \cdot g' = 2 \cdot 0 = 0$ Not equal