AP CALCULUS
DERIVATIVE TEST REVIEW

Chap 3 Review

Name Answer Key

Directions: Use these questions and answers to help you study for your test! No calculator.

1) If $f(x) = -2x^{\frac{2}{3}}$, then $f'(8) =$

$f'(x) = -\frac{4}{3}x^{-\frac{1}{3}}$

$= \frac{-4}{3\sqrt[3]{x}}$

$f'(8) = \frac{-4}{6} = -\frac{2}{3}$

A) $\frac{1}{3}$
B) $\frac{2}{3}$
C) $-\frac{8}{3}$
D) $-\frac{2}{3}$
E) $-\frac{1}{3}$

2) If $f(x) = \frac{3x-2}{2x+1}$, then $f'(x) =$

$y' = \frac{(2x+1)(3)-(3x-2)(2)}{(2x+1)^2}$

$= \frac{6x+3-6x+4}{(2x+1)^2}$

$= \frac{7}{(2x+1)^2}$

A) $\frac{3}{2}$
B) $\frac{-7}{(2x+1)^2}$
C) $\frac{12x+1}{(2x+1)^2}$
D) $\frac{12x-1}{(2x+1)^2}$
E) $\frac{7}{(2x+1)^2}$

3) If $x^2 - 2xy + y^3 = 10$, then $\frac{dy}{dx} =$

$2x - [(2x)(\frac{dy}{dx}) + (y)(2)] + 3y^2\frac{dy}{dx} = 0$

$2x - 2x\frac{dy}{dx} - 2y + 3y^2\frac{dy}{dx} = 0$

$-2x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 2y - 2x$

$\frac{dy}{dx}(-2x + 3y^2) = 2y - 2x$

$\frac{dy}{dx} = \frac{2y-2x}{3y^2-2x}$

A) $\frac{2y-2x}{3y^2+2x}$
B) $\frac{2y+2x}{3y^2+2x}$
C) $\frac{2y-2x}{3y^2-2x}$
D) $\frac{2y+2x}{3y^2-2x}$
E) $\frac{2y-2x+2}{3y^2}$

4) If $f(x) = \csc x \tan x$ ($\frac{1}{\sin x} \cdot \frac{\sin x}{\cos x}$), then $f'(x) =$

Simplify f(x) First! Then take the deriv!

$f(x) = \frac{1}{\cos x} = \sec x$

$f'(x) = \sec x \tan x$

A) $\frac{1}{\sin^2 x \cos x}$
B) $\sec x \tan x$
C) $-\csc x \cot x$
D) $-\frac{1}{\sin x}$
E) $-\sec x \tan x$

5) If $f(x) = \sin^2(2x)$, then $f'\left(\frac{\pi}{6}\right) =$ (Double Chain Rule)

$f'(x) = 2(\sin(2x))\cos(2x) \cdot 2$

$= 4\sin(2x)\cos(2x)$

$f'(\frac{\pi}{6}) = 4\sin\frac{\pi}{3}\cos\frac{\pi}{3}$

$= 4 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$

$= \sqrt{3}$

A) $\frac{\sqrt{3}}{2}$
B) $2\sqrt{3}$
C) 2
D) $\sqrt{3}$
E) 4

6) If $f(x) = \tan(2x^2)$, then $f'(\sqrt{\pi}) =$ (Single Chain Rule)

$\sec^2(2x^2) \cdot 4x$

$\sec^2(2\pi) \cdot 4\sqrt{\pi}$

$(1)^2\, 4\sqrt{\pi}$

$4\sqrt{\pi}$

A) $\sqrt{\pi}$
B) $4\sqrt{\pi}$
C) 1
D) $2\sqrt{\pi}$
E) 0

7) If $y = \frac{e^{2x}}{x+2}$, then $\frac{dy}{dx} =$

$y' = \frac{(x+2)(2e^{2x}) - (e^{2x})(1)}{(x+2)^2}$

$y' = \frac{e^{2x}(2x+4-1)}{(x+2)^2}$

$y' = \frac{e^{2x}(2x+3)}{(x+2)^2}$

A) $\frac{e^{2x}(2x+5)}{(x+2)^2}$
B) $\frac{e^{2x}}{x}$
C) $\frac{2e^{2x}}{(x+2)^2}$
D) $\frac{e^{2x}(x+1)}{(x+2)^2}$
E) $\frac{e^{2x}(2x+3)}{(x+2)^2}$

8) For differentiable functions f and g, the table below shows various values for $f(x)$ and $g(x)$ as well as their derivatives, $f'(x)$ and $g'(x)$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	3	5	4
2	0	-1	1	-3

If $H(x) = g[f(x)]$, then $H'(1) =$

$g'(f(x)) \cdot f'(x)$

$g'(f(1)) \cdot f'(1)$

$g'(2) \cdot 3$

$-3 \cdot 3$

-9

A) 0
B) -15
C) -3
D) 10
E) 20
F) -9

9) If function f is defined by $f(x) = x^5 + x - 3$, and f^{-1} represents the inverse of function f, then $(f^{-1})'(-1) =$

$f^{-1}(-1) = 1$

A) $\frac{1}{6}$

B) $-\frac{1}{6}$

C) 6

D) $-\frac{1}{5}$

E) $\frac{1}{3}$

$x = y^5 + y - 3$

$1 = 5y^4\frac{dy}{dx} + \frac{dy}{dx}$

$1 = \frac{dy}{dx}(5y^4 + 1)$

$\frac{dy}{dx} = \frac{1}{5y^4+1} = \frac{1}{5(1)^4+1} = \frac{1}{6}$

Guess + check to find the y-value.

$-1 = y^5 + y - 3$

$0 = y^5 + y - 2$

$y = 1$

10) If $f(x) = \ln(2\ln x)$, then $f'(e^2) =$

A) $\frac{2}{e^2}$

B) $\frac{1}{4e}$

C) $\frac{1}{2e^2}$

D) $\frac{1}{e^4}$

E) $2e^2$

$y = \ln(\ln x^2)$ (log property)

$y' = \frac{1}{\ln x^2} \cdot \frac{1}{x^2} \cdot 2x$

$y' = \frac{2}{x \ln x^2}$

Don't worry about evaluating at $x = e^2$

$f'(e^2) = \frac{2}{e^2 \ln e^4} = \frac{2}{e^2 \cdot 4} = \frac{1}{2e^2}$

11) If $y = 5^{x^3+1}$, then $\frac{dy}{dx} =$ (Log Diff!)

A) $(3x^2)5^{x^3+1}$

B) $5^{3x^2}(\ln 5)$

C) $3x^2(\ln 5)5^{x^3+1}$

D) $(\ln 5)5^{x^3+1}$

E) $(x^3+1)5^{x^3}$

$\ln y = \ln 5^{x^3+1}$

$\ln y = (x^3+1)(\ln 5)$

$\frac{1}{y}\frac{dy}{dx} = (x^3+1)(0) + (\ln 5)(3x^2)$

$\cancel{y}\frac{1}{y}\frac{dy}{dx} = (3x^2 \ln 5) \cdot y$

$\frac{dy}{dx} = (3x^2 \ln 5)(5^{x^3+1})$

12) If $f(x) = \sec^{-1}(x^2)$, then $f'(x) =$

A) $\frac{2}{x\sqrt{x^4-1}}$

B) $\frac{2}{x\sqrt{1-x^4}}$

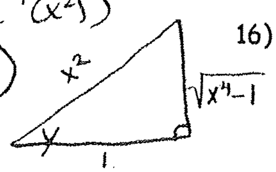
C) $\frac{2x}{x^4+1}$

D) $\frac{2x}{\sqrt{x^4-1}}$

E) $\frac{2}{x(x^2-1)}$

$\sec y = \sec(\sec^{-1}(x^2))$

$\sec y = x^2 \left(\frac{H}{A}\right)$



$\sec y \tan y \frac{dy}{dx} = 2x$

$\frac{dy}{dx} = \frac{2x}{\sec y \tan y}$

$\frac{dy}{dx} = \frac{2x}{x^2\sqrt{x^4-1}}$

$\frac{dy}{dx} = \frac{2}{x\sqrt{x^4-1}}$

13) If $y = \sin^{-1}\frac{1}{\sqrt{x}}$, then $\frac{dy}{dx} =$

A) $\frac{2}{x\sqrt{x^2-1}}$

B) $-\frac{2}{x\sqrt{x-1}}$

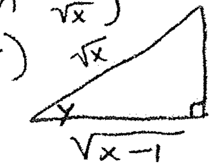
C) $-\frac{1}{2x\sqrt{x-1}}$

D) $\frac{1}{2x\sqrt{x-1}}$

E) $\frac{2x}{\sqrt{x-1}}$

$\sin y = \sin(\sin^{-1}\frac{1}{\sqrt{x}})$

$\sin y = \frac{1}{\sqrt{x}} \left(\frac{O}{H}\right)$



$\cos y \frac{dy}{dx} = \frac{-1}{2\sqrt{x^3}}$

$\frac{dy}{dx} = \frac{-1}{2\sqrt{x^3}\cos y}$

$\frac{dy}{dx} = \frac{-1}{2\sqrt{x^3} \cdot \frac{\sqrt{x-1}}{\sqrt{x}}} = \frac{-1}{2x\sqrt{x-1}}$

Deriv. of $\frac{1}{\sqrt{x}}$

$y = x^{-\frac{1}{2}}$ $y' = -\frac{1}{2}x^{-\frac{3}{2}} = \frac{-1}{2\sqrt{x^3}}$

14) If $y^2 - 2xy + 3x^3 = 5$, then at the point (-1,2), $\frac{dy}{dx} =$

A) $-\frac{13}{6}$

B) $\frac{13}{6}$

C) $-\frac{5}{6}$

D) $\frac{5}{6}$

E) $\frac{9}{2}$

$2y\frac{dy}{dx} - [(2x)(\frac{dy}{dx}) + (y)(-2)] + 9x^2 = 0$

$2y\frac{dy}{dx} - 2x\frac{dy}{dx} + 2y + 9x^2 = 0$

$2y\frac{dy}{dx} - 2x\frac{dy}{dx} = -2y - 9x^2$

$\frac{dy}{dx}(2y - 2x) = -2y - 9x^2$

$\frac{dy}{dx} = \frac{-2y-9x^2}{2y-2x}$

$\frac{dy}{dx} = \frac{-4-9}{4+2} = \frac{-13}{6}$

15) Suppose functions f and g are differentiable functions, where $f(-3) = 2$, $f'(-3) = 6$, $g(-3) = 3$, and $g'(-3) = 4$.

If $H(x) = \frac{x+g(x)}{f(x)}$, then $H'(-3) =$

A) 1

B) $\frac{3}{2}$

C) $\frac{5}{2}$

D) $-\frac{1}{2}$

E) 0

$H'(x) = \frac{(f(x))(1+g'(x)) - (x+g(x))(f'(x))}{(f(x))^2}$

$H'(-3) = \frac{f(-3)(1+g'(-3)) - (-3+g(-3))f'(-3)}{(f(-3))^2}$

$= \frac{(2)(1+4) - (-3+3)(6)}{2^2}$

$= \frac{10-0}{4} = \frac{10}{4} = \frac{5}{2}$

16) If $y = x^{e^x}$, then $\frac{dy}{dx} =$ (Log. Diff.)

A) $e^x\left(\frac{1}{x} + \ln x\right)x^{e^x}$

B) $e^x\left(\frac{1}{x} + \ln x\right)$

C) $e^x x^{(e^x-1)}$

D) $e^x x^{ex}$

E) $x^{(e^x+1)}\left(\frac{1}{x} + \ln x\right)$

$\ln y = \ln x^{e^x}$

$\ln y = e^x \ln x$

$\frac{1}{y}\frac{dy}{dx} = (e^x)(\frac{1}{x}) + (\ln x)(e^x)$

$\cancel{y}\frac{1}{y}\frac{dy}{dx} = (e^x(\frac{1}{x} + \ln x)) \cdot y$

$\frac{dy}{dx} = e^x(\frac{1}{x} + \ln x)(x^{e^x})$