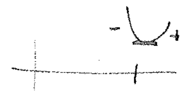
Calculus Test Chapter 4 Key

True/False



1. $f'(4) = 0$.
   $f' > 0$ to the right of $x = 4$.
   $f' < 0$ to the left of $x = 4$. Therefore, a relative minimum occurs at $x = 4$. — 1. True

2. If $f'(4) = 9$, then $f(x)$ is increasing at $x = 4$. — 2. True

3. If y' is increasing on a given interval, then y is concave up on the interval. — 3. True

4. If $y'' < 0$ on a given interval, then y is concave down on the interval. — 4. True

5. The critical points of a function occur only where $f'(x) = 0$. — 5. False

Multiple Choice

6. The function $y = f(x)$ whose first derivative $y' = (x - 2)(2x - 3)^2$ has inflection point(s) at: — 6. a

   a. 1.5, 1.833 b. 2, 1.5 c. 1.5, 5.5 d. 1.833, 5.5

7. Determine where $y = 7x^3 - x^2 - 8x + 5$ has local maximum or minimum values. — 7. c

   a. local max where $x = -4/3$
   local min where $x = 2/7$

   b. local max where $x = 2/7$
   local min where $x = -4/3$

   c. local max where $x = -4/7$
   local min where $x = 2/3$

   d. local max where $x = 2/3$
   local min where $x = -4/7$

8. Find the absolute maximum and the absolute minimum of $y = (1/5)x^5 - (4/3)x^3 + 7$ on the interval $[-2.5, 2.5]$. — 8. c

   a. absolute max of 9.266
   absolute min of 0.730

   b. absolute max of 0.733
   absolute min of -9.266

   c. absolute max of 11.2667
   absolute min of 2.7333

   d. absolute max of 7.6
   absolute min of -17.6

9. Find the value or values of c that satisfy the Mean Value Theorem for $y = x^2 - 6x + 4$ on $2 \le x \le 7$. — 9. c

   a. 7/5 b. 37/10 c. 9/2 d. 15/9

$f'(x) = 2x - 6$

$f'(c) = 2c - 6$

$f(7) = 11$

$f(2) = -4$

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$2c - 6 = \frac{11 - -4}{7 - 2}$

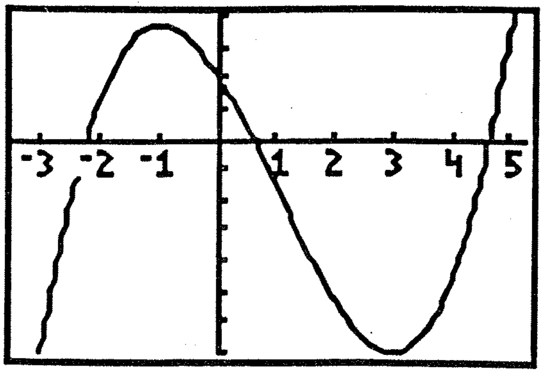
$2c - 6 = \frac{15}{5}$

$2c - 6 = 3$

$2c = 9$

$c = 9/2$

Use the graph of $y = f(x)$ given below to determine the following. Select your answers from a--m given below. [May use more than 1 letter.]



(10) estimate where $f'(x) = 0$. f, i

(11) estimate where $f'(x) > 0$. e, h

(12) estimate where $f'(x) < 0$. a

(13) estimate where $f''(x) = 0$. d

(14) estimate where $f''(x) > 0$. g

(15) estimate where $f''(x) < 0$. b

a. $-1 < x < 3$	b. $x < 1$
c. $1 < x < -3$	d. $x = 1$
e. $x < -1$	f. $x = -1$
g. $x > 1$	h. $x > 3$
i. $x = 3$	k. $x = -3$
l. $x < 3$	m. $x > -1$

(16) Find the general antiderivative for $x^4 - 8 + 6/x^7$. Support your result with a graphing utility. $+6x^{-7}$ $-x^{-6}$ (16) c

a. $x^5 - 8x + 1/x^6 + C$
b. $x^5 - 8x - 1/x^6 + C$
c. $x^5/5 - 8x - 1/x^6 + C$
d. $x^5/5 - 8x + 1/x^6 + C$

(17) The slope of a curve at the point (x,y) is $4x - 3$. Find the curve if it is required to pass through the point $(1, 1)$. (17) b

a. $2x^2 + 3x + 2$
b. $2x^2 - 3x + 2$
c. $4x^2 - 3x$
d. $4x^2 - 3x + 2$

$\frac{dy}{dx} = 4x - 3$
$y = 2x^2 - 3x + C$
$1 = 2 - 3 + C$
$2 = C$

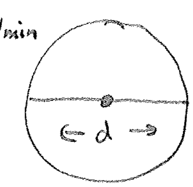
(18) A tanker is spilling oil into the Gulf of Mexico resulting in an oil slick that is close to circular in shape. At the time the slick's diameter is growing at the rate of 4 ft/min, the diameter is 500 feet. At what rate is the area of the oil slick spreading? (18) b

a. 12566.4 ft^2/min
b. 3141.6 ft^2/min
c. 1570.8 ft^2/min
d. 4000π ft^2/min

$\frac{dd}{dt} = 4$ ft/min

$d = 500$ ft.

find $\frac{dA}{dt}$



$A = \pi r^2$

$d = 2r$

$\frac{1}{2}d = r$

$A = \pi\left(\frac{1}{2}d\right)^2$

$A = \frac{\pi}{4}d^2$

$\frac{dA}{dt} = \frac{\pi}{2} d \frac{dd}{dt}$

$\frac{dA}{dt} = \frac{\pi}{2}(500)(4)$

$\frac{dA}{dt} = 1000\pi$ ft^2/min

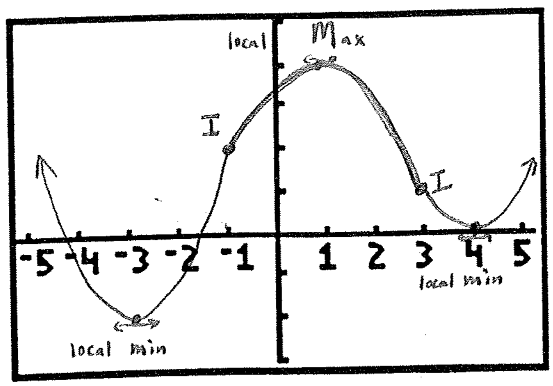
Sketch a continuous curve of $y = f(x)$ having the following characteristics.

$f(-3) = -2$
$f(-1) = 2$ f
$f(1) = 4$
$f(3) = 1$
$f(4) = 0$

$f'(-3) = 0$ f'
$f'(1) = 0$
$f'(4) = 0$
$f'(x) < 0;\ x < -3$
$f'(x) > 0;\ -3 < x < 1$
$f'(x) < 0;\ 1 < x < 4$
$f'(x) > 0;\ x > 4$

$f''(-1) = 0$ f''
$f''(3) = 0$
$f''(x) < 0;\ -1 < x < 3$
$f''(x) > 0;\ x > 3;\ x < -1$

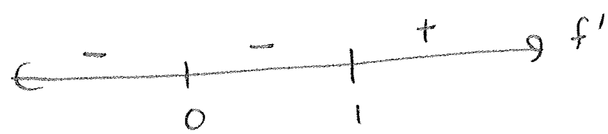
(19)



Analytically, using sign graphs for f' and f'', show where $f(x) = 3x^4 - 4x^3$:

$f'(x) = 12x^3 - 12x^2$
$12x^2(x-1)$

(20) is increasing, decreasing.



increasing $[1, \infty)$

decreasing $(-\infty, 1]$

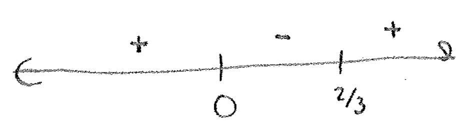
(21) is concave up, concave down.

$f''(x) = 36x^2 - 24x$
$12x(3x-2)$

$3x = 2$
$x = 2/3$

concave up $(-\infty, 0) \cup (2/3, \infty)$

concave down $(0, 2/3)$



(22) has local maxinums, local minimums, and points of inflection.

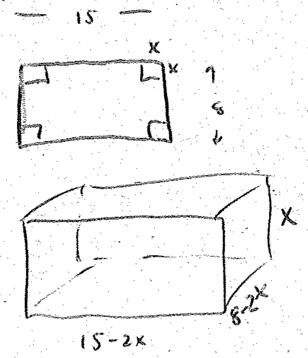
local max none

local min at $x=1$ $(1, -1)$

POI at $x = 0, 2/3$ $(0, 0)$ $(2/3, \frac{-16}{27})$

$\approx -.59259$ Decimal Approx.

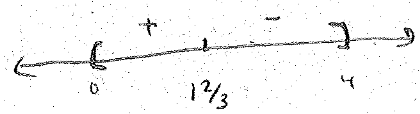
(23) You are planning to make an open rectangular box from an 8-in x 15-in piece of cardboard by cutting squares from the corners and folding up the sides. What are the dimensions of the box of largest volume you can make this way?



$V = (15-2x)(8-2x)(x)$ $\quad [0,4]$

$V = 120x - 46x^2 + 4x^3$

$V' = 120 - 92x + 12x^2$



$\boxed{11\tfrac{2}{3} \times 4\tfrac{2}{3} \times 1\tfrac{2}{3}}$

or

$\boxed{\frac{35}{3} \times \frac{14}{3} \times \frac{5}{3}}$

(23) A rectangle has its base on the x-axis and upper two vertices on $y = 12 - x^2$. Find the dimensions of the largest rectangle that can be formed.

$A = 2xy$

$A = 2x(12 - x^2)$

$A = 24x - 2x^3 \quad [0, \sqrt{12}]$

$A' = 24 - 6x^2 = 0$

$24 = 6x^2$

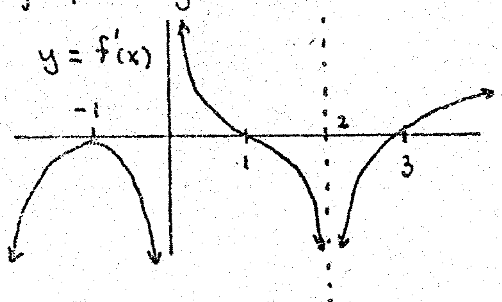
$4 = x^2$

$2 = x$

$A'' = -12x \quad A''(2) = -24 \therefore \max$

Dim 4×8

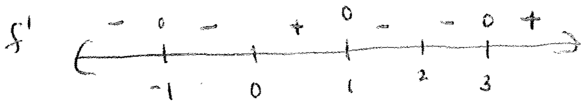
Extra Credit (2 pts) Let f be a continuous function with $f(0) = f(2) = 0$. If the graph of $y = f'(x)$ is as shown, sketch a possible graph for $y = f(x)$.

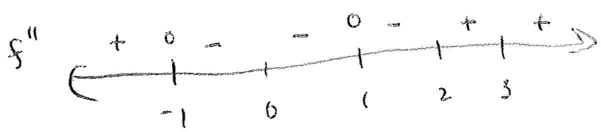


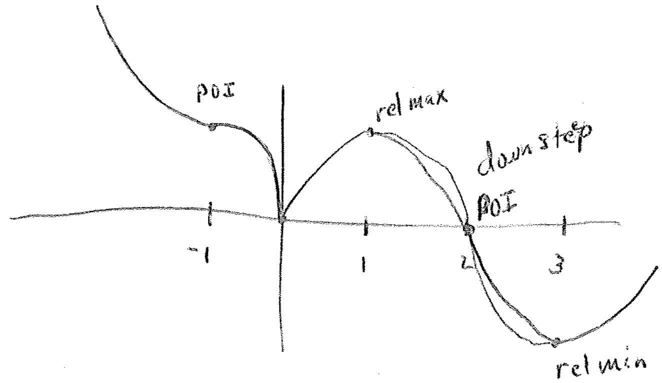
see previous page

Extra Credit (2pts) Prove that if f is continous on $[a,b]$ and $f'(x) > 0$ on (a,b) then f is increasing on $[a,b]$.

E.C. 2pts.







E.C. 2pts Let x_1 x_2 be in $[a,b]$ with $x_1 < x_2$.

Then there is a c, $x_1 < c < x_2$ such that

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Since $f'(c) > 0$ and $x_2 - x_1 > 0$ it follows that

$$f(x_2) - f(x_1) > 0.$$

$\therefore$ $f(x_2) > f(x_1)$ and f is increasing on $[a,b]$.