*sec 7
Skip 1, 2, 5, 8, 10, 11, 18

Key

Chapter Test Version A-B

True/False

1. If cost, $C(x) = 4x^3 - 7x^2 + 5x - 9$, then the average cost function is $AC(x) = 4x^2 - 7x + 5$. F

2. A company would realize a maximum profit when their marginal cost is equal to their marginal revenue. T

3. The function $f(x) = 3 + 4\cos x + \cos 2x$ is always greater than or equal to 0. T
   Graph - always above x-axis or on x-axis

4. The function $y = e^{2x+1} + 5$ is increasing and concave up. T
   $y' = e^{2x+1} \cdot 2 = 0$, $e^{2x+1} = 0$ never; y': + ; No min/max
   $y'' = e^{2x+1} \cdot 2$, $e^{2x+1} = 0$ never; y'': + ; Always concave up

5. The antiderivative of a constant is zero. F

Multiple Choice/Free Response

1. Determine where $y = 5x^3 + 4x^2 - 12x + 7$ has local maximum or minimum values.

   $y' = 15x^2 + 8x - 12 = 0$
   $(5x+6)(3x-2) = 0$
   $x = -\frac{6}{5}$ $x = \frac{2}{3}$
   f': + − + at $-\frac{6}{5}$, $\frac{2}{3}$

   a. local max where $x = -6/5$; local min where $x = 2/3$
   b. local max where $x = -2/3$; local min where $x = 6/5$
   c. local max where $x = 6/5$; local min where $x = -2/3$
   d. local max where $x = 2/3$; local min where $x = -6/5$

2. Find the absolute maximum and the absolute minimum of $y = (2/5)x^5 - (3/2)x^3 + 5$ on the interval $[-2, 2]$.

   $y' = 2x^4 - \frac{9}{2}x^2 = 0$ | $x^2(2x^2 - \frac{9}{2}) = 0$
   $x = 0$ $2x^2 = \frac{9}{2}$ $x^2 = \frac{9}{4}$ $x = \pm\frac{3}{2}$

   a. absolute max of 7.266; absolute min of - 10.831
   b. absolute max of 10.754; absolute min of - 2.366
   c. absolute max of 11.2667; absolute min of 2.7333
   d. absolute max of 7.025; absolute min of 2.975

   f': + − − + ; −2, −1.5, 0, $\frac{3}{2}$, 2
   −2 min $y = 4.2$; −1.5 max $y = 7.025$; $\frac{3}{2}$ min $y = 2.975$; 2 max $y = 5.8$

3. Find the value or values of c that satisfy the Mean Value Theorem for $y = 2x^2 - 5x + 1$ on $-2 \le x \le 3$.
   ($a = -2$, $b = 3$)

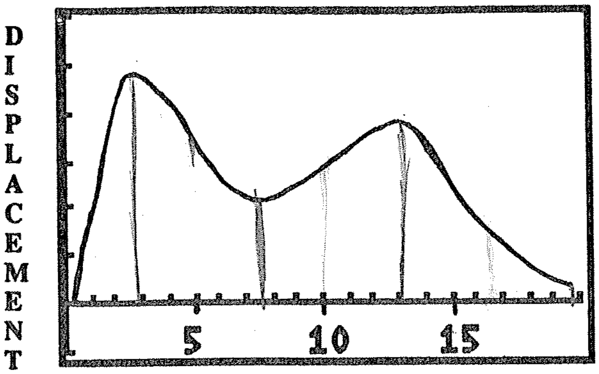
   a. 1/2 b. 2 c. 9/2 d. 9

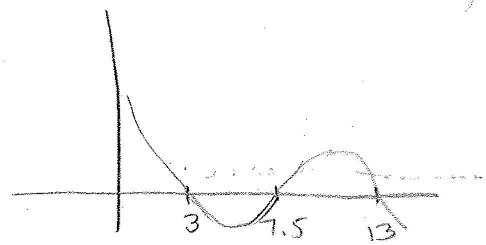
   #3
   $y' = 4x - 5$
   $f'(c) = \frac{f(b) - f(a)}{b - a}$
   $4x - 5 = \frac{f(3) - f(-2)}{3 + 2}$
   $4x - 5 = \frac{4 - 19}{5}$
   $4x - 5 = \frac{-15}{5}$
   $4x - 5 = -3$
   $4x = 2$
   $x = \frac{1}{2}$
   $c = \frac{1}{2}$

4. The graph below shows the position of a particle moving back and forth on a coordinate line.

   a. When is the velocity equal to zero? $\approx 3, \approx 7.5, \approx 13$ ($f' = 0$)

   b. When is the acceleration equal to zero? $\approx 5, \approx 10, \approx 16.5$
   where concavity changes

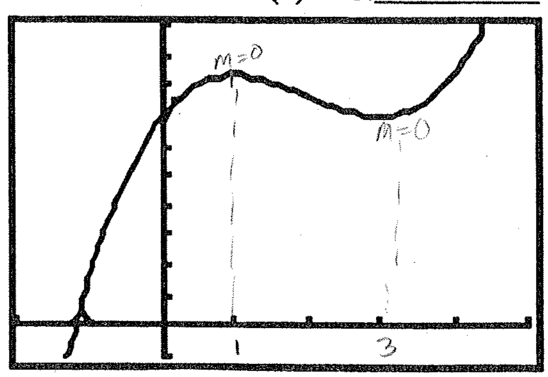




5. Use the graph of $y = f(x)$ given below to determine the following. Select your answers from a--m given below.

A. estimate where $f'(x) = 0$. i, f
B. estimate where $f'(x) > 0$. h, e
C. estimate where $f'(x) < 0$. a
D. estimate where $f''(x) = 0$. d
E. estimate where $f''(x) > 0$. g
F. estimate where $f''(x) < 0$. b

a. $1 < x < 3$	b. $x < 2$
c. $1 < x < -3$	d. $x = 2$
e. $x < 1$	f. $x = 1$
g. $x > 2$	h. $x > 3$
i. $x = 3$	k. $x = -3$
l. $x < 3$	m. $x > -1$



[- 2, 5] by [- 1, 10]

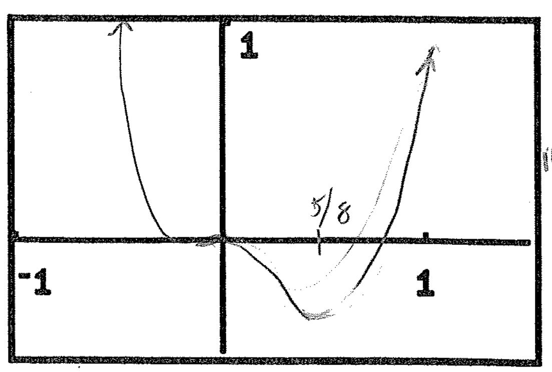
6. Draw a complete graph of $y = f(x) = 6x^4 - 5x^3$. Then prove analytically that your graph is complete by confirming the inflection points, local maximums and minimums, and the intervals upon which the function is increasing, decreasing, concave up, and concave down.

$f'(x) = 24x^3 - 15x^2$
$0 = 24x^3 - 15x^2$ $\quad f' = 0$
$0 = 3x^2(8x - 5)$
$x = 0 \quad 8x - 5 = 0$
$x = 5/8$

critical pts
f' = und never

f' − | − | + (0, 5/8)

min at $x = \frac{5}{8}$, No Max



[- 1, 1.5] by [- 0.5, 1]

increase: $[5/8, \infty)$
decrease: $(-\infty, 5/8)$

$f''(x) = 72x^2 - 30x$
inflection pts $0 = 6x(12x - 5)$
$x = 0 \quad x = \frac{5}{12}$

f'' + | − | + (0, 5/12)

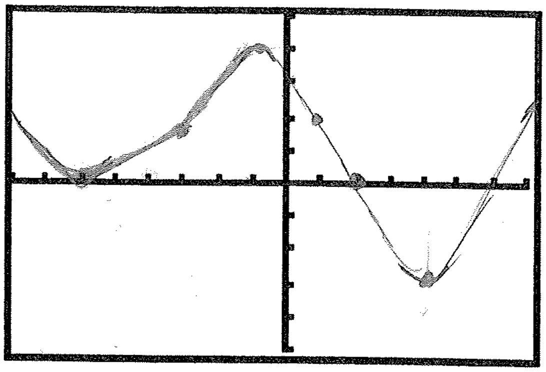
inflection pts at $x = 0$, $x = 5/12$

concave up $(-\infty, 0)(\frac{5}{12}, \infty)$ | concave down $(0, 5/12)$

7. Sketch a continuous curve of $y = f(x)$ having the following characteristics.

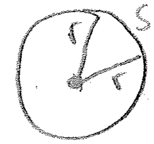
$f(-3) = 1.5$	$f'(4) = 0$	$f''(-3) = 0$
$f(-1) = 4$	$f'(-1) = 0$	$f''(1) = 0$
$f(1) = 2$	$f'(-6) = 0$	$f''(x) < 0;\ -3 < x < 2$
$f(2) = 0$	$f'(x) < 0;\ x < -6$	$f''(x) > 0;\ x > 1;\ x < -3$
$f(4) = -3$	$f'(x) > 0;\ -6 < x < -1$	
~~$f(-4) = 0$~~	$f'(x) < 0;\ -1 < x < 4$	
$f(-6) = 0$	$f'(x) > 0;\ x > 4$	

? crossoff



[-8, 7] by [- 5, 5]

#10



$2r+s=16$

$A=\pi r^2$

8. The function $f(x) = 2x^2 - 7x - 15$ has a zero between - 1 and - 2. Using $x_o = -1.2$, carry out Newton's Method three times to approximate this zero to 4 decimal places.

a. - 1.6708 b. 1.5000 c. - 1.4718 d. - 1.2673

#9

l, w

9. A rectangle has a perimeter of 100 feet. Find the dimensions of this rectangle that will maximize the area.

$2l+2w=100$
$l+w=50$
$l=50-w$

a. l = 10; w = 40 b. l = 5; w = 45 c. l = 20; w = 30 d. l = 25; w = 25

$A=l\cdot w$
$A=(50-w)(w)$
$A=50w-w^2$
$A'=50-2w=0$
$2w=50$
$w=25$
$l=25$

10. A sector with a perimeter of 16 is to be cut out a circle. If r is the radius and s is the measure of the arc length, what values of r and s (select answers from a - d below) will maximize the area of the sector.

$r=\frac{16}{3}$ $s=\frac{16}{3}$

a. 4 b. 6 c. 7 d. 8

11. A company shows a revenue of $r(x) = 60x$, where x is the number of whole units produced. The cost for producing these x units is $C(x) = 3x^3 - \frac{19x^2}{2} + 28x$.

How many units must be produced to maximize profit ? e

What is the maximum profit ? b

a. 183 b. 352 c. 10 d. 25

e. 8 f. 533 g. 3 h. 30

$A=100$ $\ell = \frac{100}{w}$ $P = 2\ell + 2w$ $P = 2(\frac{100}{w}) + 2w$ $P = \frac{200}{w} + 2w$ $P' = \frac{-200}{w^2} + 2$

12. What is the smallest perimeter possible for a rectangle whose area is to be 100 square inches?
   a. 20 b. 30 c. 40 d. 50

$2 = \frac{200}{w^2}$ $2w^2 = 200$ $w^2 = 100$ $w = \sqrt{100} = 10$ $\ell = 10, w = 10$ $\underline{P = 40}$

13. The sum of two nonnegative numbers is 25. The sum of the squares of the number is to be as large as possible. Find ~~the two numbers~~.
   a. 10, 15 b. 12.5, 12.5 c. 17, 18 d. 20, 5

$x + y = 25$ $y = 25 - x$ $x^2 + (25-x)^2 = f(x)$ $2x + 2(25-x)(-1) = f'(x)$ $2x + -50 + 2x = f'(x)$ $4x - 50 = 0$ $4x = 50$ $x = 12.5$

14. The function $y = f(x)$ whose first derivative is $y' = (x - 5)(3x - 2)^2$ has inflection point(s) at $x =$:
   a. 1.5, 1.833 b. 2/3, 28/9 c. 2/3, 32/9 d. 1.833, 5.5

$y'' = 1 \cdot (3x-2)^2 + (x-5)2(3x-2)3$ $= 9x^2 - 12x + 4 + 18x^2 - 102x + 60$ $= 27x^2 - 114x + 64$ $x = \frac{114 \pm \sqrt{114^2 - 4(27)(64)}}{2(27)}$ $x = 3.5$ $x = .6$

15. A ball is thrown into the air along a path described by $y = \frac{-x^2}{80} + x$, where y is its height in feet above the ground. The ball reaches a maximum height of __a__ ft and comes to rest at __f__ ft from where it is thrown.
   a. 20 b. 30 c. 40 d. 60
   e. 70 f. 80

$0 = \frac{-x^2}{80} + x = x(-\frac{x}{80} + 1)$ $x = 0$ $\frac{-x}{80} = -1$ $\boxed{x = 80}$ $y' = \frac{-x}{40} + 1 = 0$ $1 = \frac{x}{40}$ $x = 40$ $\frac{-40^2}{80} + 40 = \boxed{20}$

16. A chemical substance is spreading in a nearly circular shape on the surface of the water in a large holding tank pool. At the time the radius of the chemical is increasing at a rate of 1 ft/minute, the diameter is 10 feet. At what rate is the area of the chemical spreading?
   a. 10 ft²/min b. 31.4159 ft²/min
   c. 65.7018 ft²/min d. 40π ft²/min

$\frac{dr}{dt} = 1$ ft/min $\frac{dA}{dt} = ?$ $r = 5$ ft $A = \pi r^2$ $\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} = 2\pi(5)(1) = \boxed{10\pi}$

17. You are letting the air out of a hot air spherical balloon at a steady rate. The volume of the balloon is decreasing at the rate of 15π m³/min when the diameter is 70 meters. At what rate is the radius of the balloon changing?
   a. -.0031 m/min b. - .0051 m/min
   c. - .0083 m/min d. - .0213 m/min

$\frac{dr}{dt} = ?$ $\frac{dV}{dt} = -15\pi$ m³/min $r = 35$ m $V = \frac{4}{3}\pi r^3$ $\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$ $-15\pi = 4\pi(35)^2 \frac{dr}{dt}$ $\frac{-15}{4900} = \frac{dr}{dt}$ $-.0031 = dr/dt$

18. Find the general antiderivative for $3x^4 + 7x + 6/x^4$. Support your result with a graphing utility.
   a. $x^5 + 7x + 2/x^3 + C$ b. $3x^5 + 7x - 2/x^3 + C$
   c. $x^5/5 + 7x - 2/x^3 + C$ d. $3x^5/5 + 7x^2/2 - 2/x^3 + C$

19. The slope of a curve at the point (x,y) is $5x - 7$. Find the curve if it is required to pass through the point (1, 1).
   a. $5x^2 + 7x + 2$ b. $5x^2 - 7x - 3$
   c. $5x^2/2 - 7x + 11/2$ d. $5x^2 - 7x + 5$

$\frac{5x^2}{2} - 7x + C$ $1 = \frac{5(1)^2}{2} - 7(1) + C$ $1 = \frac{5}{2} - \frac{14}{2} + C$ $1 + \frac{9}{2} = C$ $\frac{11}{2} = C$

20. The marginal cost of manufacturing an item when x thousand items are produced is $\frac{dC}{dx} = 5x^3 - 4x + 8$ dollars per thousand. Find the cost function $C(x)$ if $C(0) = 355$.
   a. $4x^4/4 - 4x^2 + 8x + 355$ b. $5x^4/4 - 2x^2 + 8x - 355$
   c. $4x^4 - 6x^2 + 5x + 550$ d. $5x^4 - 2x^2 + 8x + 355$

$\frac{5x^4}{4} - \frac{4x^2}{2} + 8x + C = \frac{5x^4}{4} - 2x^2 + 8x + C$ $355 = \frac{5(0)^4}{4} - 2(0)^2 + 8(0) + C$ $355 = C$