Review - Chapter Test

Name Key

P. 373-375 1-8, 11-13, 15, 17, 25, 27, 31, 37, 51, 57

Solve the following integrals. Circle your final answer.

1. $\int x\sqrt{25 + x^2}\, dx$

$u = 25 + x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int u^{1/2}du = \frac{1}{2}u^{3/2}\cdot\frac{2}{3} + C$

$= \frac{1}{3}(25+x^2)^{3/2} + C$

2. $\int(3x + 5)^2\, dx$

$u = 3x+5$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$\frac{1}{3}\int u^2 du$

$\frac{1}{3}\frac{u^3}{3} + C$

$\frac{1}{9}(3x+5)^3 + C$

3. $\int \frac{x^2 + 3x}{x}\, dx$

$\int\left(\frac{x^2}{x} + \frac{3x}{x}\right)dx = \int(x+3)\,dx$

$\frac{x^2}{2} + 3x + C$

4. $\int 2x \sin(1 - x^2)\, dx$

$u = 1 - x^2$

$du = -2x\,dx$

$-du = 2x\,dx$

$-\int \sin u\, du$

$\cos u + C$

$\cos(1-x^2) + C$

5. $\int \frac{dx}{x\sqrt{x}} = \int \frac{dx}{x^{3/2}}\, dx$

$x^1 \cdot x^{1/2}$

$\int x^{-3/2} dx$

$\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + C = \frac{-2}{\sqrt{x}} + C$

6. $\int(\cos x)(7 + \sin x)^2\, dx$

$u = 7 + \sin x$

$du = \cos x\, dx$

$\int u^2 du$

$\frac{u^3}{3} + C = \frac{(7+\sin x)^3}{3} + C$

7. $\int 3x\, e^{x^2}\, dx$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$3\int e^u \cdot \frac{1}{2}du$

$\frac{3}{2}e^u + C = \boxed{\frac{3}{2}e^{x^2} + C}$

8. $\int x \cos(3 - 3x^2)\, dx$

$u = 3 - 3x^2$

$du = -6x\,dx$

$-\frac{1}{6}du = x\,dx$

$-\frac{1}{6}\int \cos u\, du$

$-\frac{1}{6}\sin u + C = \boxed{-\frac{1}{6}\sin(3-3x^2) + C}$

9. $\int 9 - \dfrac{3}{x^3}\, dx$

$\int 9\,dx - \int 3x^{-3}\,dx$

$9x - \dfrac{3x^{-2}}{-2} + C$

$\boxed{9x + \dfrac{3}{2x^2} + C}$

10. $\int 3(x + 3)(x^2 + 6x)^3\, dx$

$u = x^2 + 6x$

$du = 2x + 6\,dx$

$du = 2(x+3)\,dx$

$\frac{1}{2}du = x+3\,dx$

$\frac{1}{2} \cdot 3\int u^3\, du$

$\frac{3}{2}\frac{u^4}{4} + C$

$\boxed{\frac{3}{8}(x^2+6x)^4 + C}$

11. $\int \dfrac{x^3}{\sqrt{1 + x^4}}\, dx$

$u = 1 + x^4$

$du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

$\frac{1}{4}\int u^{-1/2}\, du$

$\frac{1}{4}\dfrac{u^{1/2}}{1/2} + C = \frac{1}{4} \cdot \frac{2}{1}(1+x^4)^{1/2} + C$

$\frac{1}{2}(1+x^4)^{1/2} + C$

12. $\int(-\sec^2 x + 4\cos x)\,dx$

$\int -\sec^2 x\,dx + \int 4\cos x\,dx$

$-\tan x + 4\sin x + C$

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① $\int_0^{\pi/3} \sec^2\theta \, d\theta = \tan\theta \Big|_0^{\pi/3} = \sqrt{3}$

② $\int_1^2 \left(x + \frac{1}{x^2}\right) dx = \frac{x^2}{2} + \frac{x^{-1}}{-1} \Big|_1^2 = 2$

③ $\int_0^1 \frac{36}{(2x+1)^3} dx$ $u = 2x+1$, $du = 2dx$, $\frac{1}{2} du = dx$ $\frac{1}{2} \cdot 36 \int u^{-3} du = \frac{18u^{-2}}{-2} = \frac{-9}{(2x+1)^2} \Big|_0^1$

$= 8$

④ $\int_{-1}^1 2x\sin(1-x^2) dx$ $u = 1-x^2$
$du = -2x\,dx$ $-\int \sin u \, du$
$-du = 2x\,dx$

$= \cos(1-x^2) \Big|_{-1}^1$

$= 0$

⑤ $\int_0^{\pi/2} 5\sin^{3/2} x \cos x \, dx$ $u = \sin x$
$du = \cos x \, dx$

$5 \int u^{3/2} du$

$5 \cdot u^{5/2} \cdot \frac{2}{5} = 2(\sin x)^{5/2} \Big|_0^{\pi/2} = 2$

(6) $\int_{1/2}^{4} \frac{x^2+3x}{x}\,dx = \int_{1/2}^{4} (x+3)\,dx$

$$= \frac{x^2}{2} + 3x \Big|_{1/2}^{4} = 147/8$$

(7) $\int_0^{\pi/4} e^{\tan x} \sec^2 x\,dx$ $\quad u = \tan x$

$du = \sec^2 x\,dx$

$\int e^u\,du$

$e^u = e^{\tan x} \Big|_0^{\pi/4} = \boxed{e-1}$

(8) $\int_1^e \frac{\sqrt{\ln r}}{r}\,dr$ $\quad u = \ln r$

$du = \frac{1}{r}dr$

$\int u^{1/2}\,du = \frac{2}{3}u^{3/2} = \frac{2}{3}(\ln r)^{3/2} \Big|_1^e = \frac{2}{3}$

(11) $\int \frac{\cos x}{2-\sin x}\,dx$ $\quad u = 2-\sin x$

$du = -\cos x\,dx$

$-\int \frac{1}{u}\,du = \boxed{-\ln|2-\sin x| + C}$

(12) $\int \frac{dx}{\sqrt[3]{3x+4}}$ $\quad u = 3x+4$

$du = 3dx$

$\frac{1}{3}du = dx$

$\frac{1}{3}\int u^{-1/3}\,du$

$\frac{1}{3}u^{2/3} \cdot \frac{3}{2} + C = \boxed{\frac{1}{2}(3x+4)^{2/3} + C}$

13. $\int \frac{t\,dt}{t^2+5}$

$u = t^2+5$
$du = 2t\,dt$
$\frac{1}{2}du = t\,dt$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\boxed{\frac{1}{2}\ln|t^2+5| + C}$

* 15. $\int \frac{\tan(\ln y)}{y}\,dy$

$u = \ln y$
$du = \frac{1}{y}\,dy$

$\int \tan u\,du = \int \frac{\sin u}{\cos u}\,du$

$v = \cos u$
$dv = -\sin u\,du$
$-dv = \sin u\,du$

$= -\int \frac{1}{v}\,dv$

$= -\ln|v| + c = -\ln|\cos u| + c$
$= \boxed{-\ln|\cos(\ln y)| + C}$

17. $\int \frac{dx}{x\ln x} =$

$u = \ln x$
$du = \frac{1}{x}\,dx$

$\int \frac{1}{u}\,du = \ln|u| + c = \boxed{\ln|\ln x| + C}$

(25) $\frac{dy}{dx} = 1 + x + \frac{x^2}{2} \quad y(0)=1$

$\int \left(1 + x + \frac{x^2}{2}\right)dx = x + \frac{x^2}{2} + \frac{x^3}{6} + C = y$

$0 + 0 + 0 + C = 1 \quad C = 1$

$\boxed{x + \frac{x^2}{2} + \frac{x^3}{6} + 1 = y}$

(27) $\frac{dy}{dx} = \frac{1}{t+4} \quad y(\overset{t}{-3}) = \overset{y}{2}$

$\int \frac{1}{t+4}\,dt \qquad u = t+4 \quad du = dt \qquad \boxed{\ln|t+4| + 2 = y}$

$\int \frac{1}{u}\,du = \ln|u| + C = y$

$\ln|t+4| + C = y$

$\ln|-3+4| + C = 2$

$0 + C = 2$

(31) $\frac{dy}{dx} = y + 2 \quad y(0) = 2$

$\int \frac{1}{y+2}\,dy = \int dx$

$u = y+2$

$du = dy$

$\int \frac{1}{u}\,du = x + C$

$\ln|y+2| = x + C$

$\ln|2+2| = 0 + C$

$\ln 4 = C$

$\ln|y+2| = x + \ln 4$

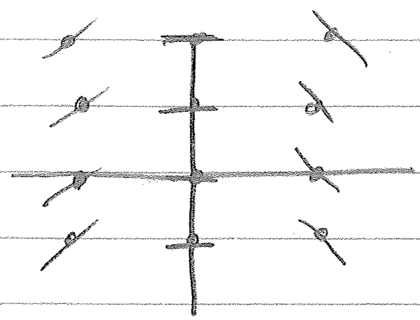
$\ln|y+2| - \ln 4 = x$

$\ln \frac{y+2}{4} = x$

$e^x = \frac{y+2}{4}$

$\boxed{4e^x - 2 = y}$

(37.)



$$\frac{dy}{dx} = -x$$

$m=0$

$X=0$

(51.) a) $y = ae^{Kt}$ $\quad .5 = 1e^{(K*2.645)}$

$\ln .5 = 2.645K$

$\boxed{.26205 \approx K}$ $(-)$ because decay

b) Skip

(57.) $$\frac{dL}{dx} = -KL$$

$$\int \frac{1}{L}\,dL = \int -K\,dx$$

$$\ln|L| = -KX + C$$

$$e^{-KX+C} = L$$

$$e^{-KX} \cdot e^{C} = L \qquad e^{C} = L_0$$

$$L = L_0 e^{-KX}$$

$$\frac{1}{2} = 1e^{(-K(18))} \rightarrow \ln\frac{1}{2} = -18K$$

$$\frac{\ln(.5)}{-18} = K = .038508$$

(1/10)

$$\frac{1}{10} = 1e^{(-.038508X)}$$

$$\ln(.1) = -.038508X \qquad X = 59.8\text{ ft}$$