Name Key

Inflection Points and Concavity

For each function below:

(a) Find the intervals of increase or decrease.

(b) Find the local maximum and minimum values.

(c) Find the intervals of concavity and the inflection points.

(d) Use the information from parts (a) - (c) to sketch the graph of f. Check your work with your graphing calculator.

1. $f(x) = 2x^3 - 3x^2 - 12x$

$f'(x) = 6x^2 - 6x - 12$

$f'(x) = 6(x^2 - x - 2)$

$= 6(x-2)(x+1)$

f' = und Never

f'' = und Never

$f''(x) = 12x - 6$

$= 6(2x-1)$

$x = \frac{1}{2}$

f': + | −1 | − | 2 | +

f'': − | ½ | +

a) increase: $(-\infty, -1]$ $[2, \infty)$

decrease: $[-1, 2]$

b) Local Max: $(-1, 2)$

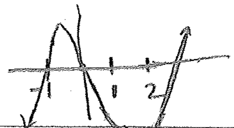
Local Min: $(2, -20)$

c) concave up: $(\frac{1}{2}, \infty)$

Concave down: $(-\infty, \frac{1}{2})$

inflection pt: $x = \frac{1}{2}$

d)



2. $f(x) = x^4 - 6x^2$

$(\sqrt{3})^4 - 6(\sqrt{3})^2$

f' = und - none

$f' = 4x^3 - 12x$

$f' = 4x(x^2 - 3)$

$x = 0$ $x = \pm\sqrt{3}$

f': − | $-\sqrt{3}$ | + | 0 | − | $\sqrt{3}$ | +

a) increasing: $[-\sqrt{3}, 0]$

$[\sqrt{3}, \infty)$

decreasing: $(-\infty, -\sqrt{3}]$

$[0, \sqrt{3}]$

b) Local Max: $(0, 0)$

Min: $(\sqrt{3}, -9)$

Abs. < $(-\sqrt{3}, -9)$

f'' = und never

$f'' = 12x^2 - 12$

$0 = 12(x^2 - 1)$

$x = \pm 1$

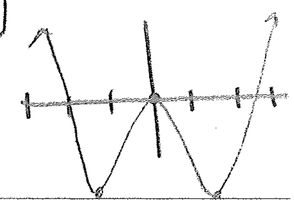
f'': + | −1 | − | 1 | +

c) Concave up: $(-\infty, -1)$

$(1, \infty)$

Concave down: $(-1, 1)$

inflection Pt: $x = \pm 1$



3. $f(x) = 2 + 3x - x^3$

$f' = 3 - 3x^2$ f' = und never

$0 = 3 - 3x^2$

$3 = 3x^2$

$x^2 = 1$

$x = \pm 1$

f': − | −1 | + | 1 | −

a) incr: $[-1, 1]$

decr: $(-\infty, -1]$

$[1, \infty)$

b) Min: $(-1, 0)$

Max: $(1, 4)$

f'' = und Never

$f'' = -6x$

$0 = -6x$

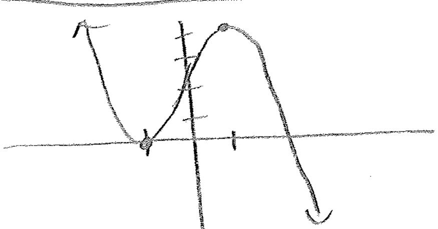
$x = 0$

f'': + | 0 | −

c) concave up: $(-\infty, 0)$

Concave down: $(0, \infty)$

inflection Pt: $x = 0$



4. $h(x) = 200 + 8x^3 + x^4$

h' = und never

$h'(x) = 24x^2 + 4x^3$

$0 = 4x^2(6 + x)$

$x = 0$ $x = -6$

f': − | −6 | + | 0 | +

a) increase: $[-6, \infty)$

decr: $(-\infty, -6]$

b) Min: $(-6, -232)$

$h'' = 48x + 12x^2$

$= 12x(4 + x)$

$x = 0$ $x = -4$

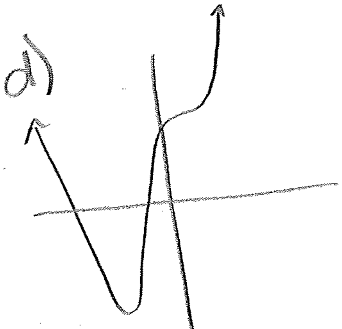
f'': + | −4 | − | 0 | +

c) Concave up: $(-\infty, -4)$

$(0, \infty)$

Concave down: $(-4, 0)$

d)



5. $k(x) = 3x^5 - 5x^3 + 3$

$K' =$ und - never
$K' = 15x^4 - 15x^2$
$0 = 15x^2(x^2 - 1)$
$x = 0 \quad x = \pm 1$

K': + | - | - | + at -1, 0, 1

a) Incr: $(-\infty, -1]$, $[1, \infty)$
decr: $[-1, 1]$

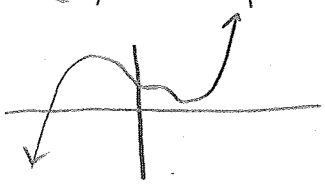
b) Min: $(1, 1)$
Max: $(-1, 5)$

$K'' =$ und - never
$K'' = 60x^3 - 30x$
$0 = 30x(2x^2 - 1)$
$x = 0 \quad 2x^2 = 1$
$x^2 = \frac{1}{2}$
$x = \pm\sqrt{.5}$

f'': - | + | - | + at $-\sqrt{.5}$, 0, $\sqrt{.5}$

c) Concave up: $(-\sqrt{.5}, 0)$, $(\sqrt{.5}, \infty)$
Concave down: $(-\infty, -\sqrt{.5})$, $(0, +\sqrt{.5})$

d)



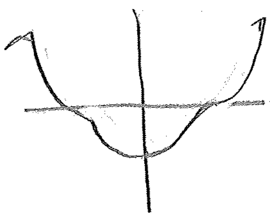
6. $f(x) = (x^2 - 1)^3$

$f' =$ und → never
$f' = 3(x^2-1)^2 \cdot 2x = 6x(x^2-1)^2$
$f' = 6x(x^2-1)(x^2-1)$
$x = 0 \quad x = \pm 1$

f': - | - | + | + at -1, 0, 1

a) incr: $[0, \infty)$
decr: $(-\infty, 0]$

b) Min: $(0, -1)$
Max: None



$f'' =$ und never
$f'' = 6(x^2-1)^2 + 6x \cdot 2(x^2-1) \cdot 2x$
$= 6(x^2-1)^2 + 24x^2(x^2-1)$
$= (x^2-1)(6(x^2-1) + 24x^2)$
$= (x^2-1)(6x^2 - 6 + 24x^2)$
$= (x^2-1)(30x^2 - 6)$
$x = \pm 1 \qquad 30x^2 = 6$
$x^2 = \frac{1}{5}$
$x = \pm\sqrt{.2}$

f'': + | - | + | - | + at -1, $-\sqrt{.2}$, $\sqrt{.2}$, 1

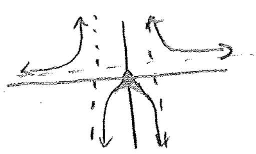
c) concave up: $(-\infty, -1)$, $(-\sqrt{.2}, \sqrt{.2})$
concave down: $(-1, -\sqrt{.2})$ $(\sqrt{.2}, 1)$

For each of the functions below:

(a) Find the vertical and horizontal asymptotes.
(b) Find the intervals of increase or decrease.
(c) Find the local maximum and minimum values.
(d) Find the intervals of concavity and the inflection points.
(e) Use the information from parts (a) - (d) to sketch the graph of f.

Skip ~~(d)~~ ~~(e)~~

7. $f(x) = \frac{x^2}{x^2 - 1}$



a) Vert: $x^2 - 1 = 0$ $\boxed{x = \pm 1}$
Horiz: $n = m$ $\boxed{y = 1}$

b) $f' = \frac{(x^2-1)(2x) - x^2(2x)}{(x^2-1)^2}$

$= \frac{2x^3 - 2x - x^3}{(x^2-1)^2} = \frac{+x^3 + 2x}{(x^2-1)^2}$

$x^3 + 2x = 0 \quad x(x^2+2) = 0$
$x = 0 \quad x$ ✗

f': + | - at 0

b) incr: $(-\infty, -1)$ $(-1, 0]$
decr: $[0, 1)$ $(1, \infty)$

c) Max: $(0, 0)$

8. $f(x) = \frac{x^2}{(x-2)^2}$

a) $(x - 2) = 0$
$\boxed{x = 2}$ Vert:
Horizontal: $\boxed{y = 1}$

$f'(x) = \frac{(x-2)^2 \cdot 2x - 2(x-2) \cdot x^2}{(x-2)^4}$

$f'(x) = \frac{(x^2 - 4x + 4)2x - 2x^3 + 4x^2}{(x-2)^4}$

$f'(x) = \frac{2x^3 - 8x^2 + 8x - 2x^3 + 4x^2}{(x-2)^4} = \frac{-4x^2 + 8x}{(x-2)^4}$

b) $-4x^2 + 8x = 0$
$-4x(x - 2) = 0$
$x = 0 \quad x = 2$

f': - | + | - at 0, 2

incr: $[0, 2)$
decr: $(-\infty, 0]$ $(2, \infty)$

c) Min: $(0, 0)$
No Max