/ Sem 1 Final Review

Semester 1 Final Exam

Name Key

Form A

Calculators Allowed

True / False: Shade in A for true and B for false

1) The $\lim_{x \to c} f(x)$ can exist even if f(c) is undefined. True

2) If f(x) is a polynomial function, then $\lim_{x \to c} f(x) = f(c)$ always. True

3) If f is a function with $f'(5) = 8$, then f must be continuous at x = 5. True

$m = 8$

4) If $f(x) = 2x - 3$ and $g(x) = x + 2$, then $g(f(4)) = 7$. True

$2x - 3 + 2 = 2x - 1 \quad 2 \cdot 4 - 1 = 7$

5) The function $f(x) = |x+2|$ is differentiable everywhere. False

corner

6) The derivative of the product of two functions is equal to the product of the derivative of those functions. False

7) The tangent to the curve $y = \sqrt{x}$ at the point x = 4 crosses the x-axis at the point (-4, 0). True

$y' = \frac{1}{2}x^{-1/2} \Big| \; y = \frac{1}{2\sqrt{4}} = \frac{1}{4} \quad (4,2) \quad m = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x-4)$

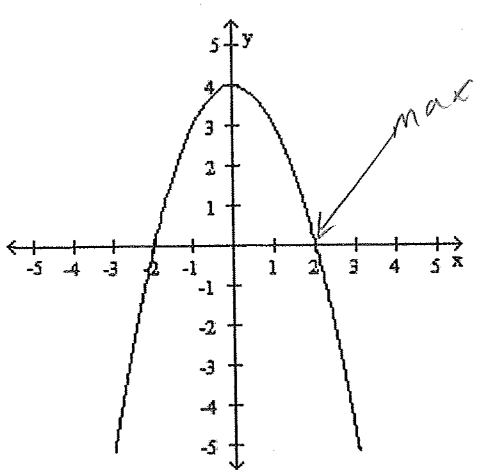
$0 - 2 = \frac{1}{4}(-4-4)$

$-2 = \frac{1}{4} \cdot -8$

$-2 = -2$

8) The function $y = 5 + e^{2x+1}$ is increasing and concave up everywhere. True

9) If the following is a graph of $f'(x)$, then the graph of f(x) has a maximum at (0, 4). False



f' switches from (+) to (-)

10) The function $f(x) = \frac{1}{x^2}$ is concave up on its entire domain. True

x^{-2} $f' = -2x^{-3}$ $f'' = 6x^{-4} = \frac{6}{x^4}$ + + 0

11) If $f'(x) = x + 2$, then $f(x)$ has a maximum at $x = -2$

12) If $f(x)$ is a continuous function on the open interval $[-3, 7]$ and $f(-1) = 4$, $f(2) = -5$ and $f(6) = 8$ then $f(x)$ has at least two zeros.

True

13) If $f(x) = \begin{cases} x^2 + 5, & \text{if } x < 2 \\ 7x - 5, & \text{if } x \geq 2 \end{cases}$, for all real numbers x, then f(x) is continuous everywhere. True

$\leftarrow 2x = 2(2) = 4 = m$

14) If $f(x) = \begin{cases} x^2 + 5, & \text{if } x < 2 \\ 7x - 5, & \text{if } x \geq 2 \end{cases}$, for all real numbers x, then f(x) is differentiable everywhere. False

$m = 7$ Cusp since $m = 4 \neq 7$

15) If $f(x) = \begin{cases} x^2 + 5, & \text{if } x < 2 \\ 7x - 5, & \text{if } x \geq 2 \end{cases}$, for all real numbers x, then f(x) has a local minimum at x = 2. False

min at $x = 0$

Multiple Choice: Choose the best choice.

16) Find $\lim_{x \to 0} \frac{2\sin 3x - 7\cos x}{3x}$ Graph in radians

A) -1.66667 B) $\frac{2}{3}$ C) 5.372471 D) does not exist

17) If $f(x) = \frac{2 + x}{3 - x}$, compute $f'(2)$. $\frac{(3-x)1 - (2+x)(-1)}{(3-x)^2} = \frac{(3-2)(1) + 2 + 2}{(3-2)^2} = \frac{5}{1}$

A) 0 B) 5 C) 2 D) 4

18) Find the x- and y- intercepts of the line that is tangent to the curve $y = x^3 - 3x^2$ at the point $(-1, -4)$.

A) $(0, -1)$ and $(-4, 0)$ B) $(0, -8)$ and $(2, 0)$ C) $(0, 5)$ and $(-\frac{6}{5}, 0)$ D) $(0, 5)$ and $(-\frac{5}{9}, 0)$

$y' = 3x^2 - 6x$
$y' = 3x(x-2) = 3(-1)(-1-2) = (-3)(-3) = 9 = m$

$y + 4 = 9(x + 1)$
$x = 0$ $y = 5$

$y + 4 = 9$
$y = 5$

$y = 0$ $x = -\frac{5}{9}$
$0 + 4 = 9x + 9$
$-5 = 9x$
$x = -\frac{5}{9}$

19) The velocity of an object at time t can be described by $v = \frac{t^2 \cos(2t + 3)}{t^2 + 4t + 1}$.

Use your graphing calculator to approximate the acceleration at time t = 2.75

A) -.057 B) -0.673 C) -0.982 D) -1.0027

-enter in y =
- 2nd calc, #6 y = 2.75

enter in y =, 2nd calc #6
x = 4, m = -1.813

20) Find the line that is tangent to the curve $y = \cos\left(\frac{2\pi x}{3}\right) + 1$ at $x = 4$

$\cos\left(\frac{8\pi}{3}\right)$ $\quad y = \frac{1}{2}$

A) $y = -1.81x - 4$ B) $y = x - 4$ C) $y = -1.81x + 7.74$ D) $y = 4x - 4$

$y - \frac{1}{2} = -1.81(x-4) \mid y = -1.81x + 7.74$

21) Suppose functions f and g and their derivatives have the following values at x = 3

$f(3) = 4$ $\quad f'(3) = \frac{3}{2}$ $\quad g(3) = 5$ $\quad g'(3) = -3$

Find the value of $\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right)$ when $x = 3$. $\quad \frac{f(x)g'(x) - g(x)f'(x)}{(f(x))^2} = \frac{4(-3) - 5\left(\frac{3}{2}\right)}{16} = \frac{-12 - \frac{15}{2}}{16}$

A) $\frac{23}{42}$ B) $-\frac{39}{32}$ C) $-\frac{13}{24}$ D) $-\frac{31}{24}$

$= -\frac{39}{2} \cdot \frac{1}{16}$

22) Find the derivative of $y = -2\sin x - 3\sec^2 x$.

A) $2\sin x - 3(\sec^2 x)(\tan x)$ B) $-2\cos x - 6(\sec^2 x)(\tan x)$

C) $\sin x - 3\sec^2 x$ D) $-2\cos x - 6\sec^2 x$

$y' = -2\cos - 6\sec x \cdot \sec x \cdot \tan x$

23) The slope of the tangent to $f(x) = \sqrt{3}\cos x$ at $x = \pi$ is

A) 0 B) 1.73 $\quad f' = 0$ C) 3 D) 3.73

24) Find $\frac{dy}{dx}$ if $y = \frac{(3x+5)^4 - 9x}{5}$. $= \frac{(3x+5)^4}{5} - \frac{9x}{5}$ $\quad f'(x) = \frac{4(3x+5)^3}{5} \cdot 3 - \frac{9}{5}$

A) $\frac{12(3x+5)^4 - 9x}{5}$ B) $12(3x+5)^3 - 9$ C) $12(3x+5)^3$ D) $\frac{12(3x+5)^3 - 9}{5}$

25) Find an equation for the tangent line to $2x^2 + 2x^2y^3 - y - 10 = 0$ at the point (1, 1).

A) $8x - 13y = 5$ B) $8x + 13y = 5$ C) $8x + 5y = 13$ D) $8x - 5y = 13$

$4x + 4x \cdot y^3 + 2x^2 \cdot 3y^2\frac{dy}{dx} - \frac{dy}{dx} = 0 \mid \frac{dy}{dx}(2x^2(3y^2) - 1) = \frac{-4x - 4xy^3}{2x^2 \cdot 3y^2 - 1}$ $\quad m = \frac{-4-4}{6-1} = -\frac{8}{5}$

26) Determine where $y = 5x^3 + 4x^2 - 12x + 7$ has local maxium or minimum values.

Graph + calc min/max

A) max at $x = -\frac{6}{5}$ min at $x = \frac{2}{3}$ B) max at $x = -\frac{2}{3}$ min at $x = \frac{6}{5}$ C) max at $x = \frac{6}{5}$ min at $x = -\frac{2}{3}$ D) max at $x = \frac{2}{3}$ min at $x = -\frac{6}{5}$

$x = -\frac{1}{2}$ $\quad x = .6$

27) Find the absolute maximum and the absolute minimum of $y = \frac{2}{5}x^5 - \frac{3}{2}x^3 + 5$ on the interval $[-2, 2]$.

A) max = 7.266 min = -10.831 B) max = 10.754 min = -2.366 C) max = 11.2667 min = 2.7333 D) max = 7.025 min = 2.975

Graph + calc min/max

$5\left[(y-1) = -\frac{8}{5}(x-1)\right]$

$5y - 5 = -8(x-1)$

$8x + 5y = 13$

$y' = 4x - 5 = -3$
$4x = 2$
$x = \frac{1}{2}$

28) Find the value of c that satisfies the Mean Value Theorem for $y = 2x^2 - 5x + 1$ on $-2 \le x \le 3$.

A) $\frac{1}{2}$ B) 2 C) $\frac{9}{2}$ D) 9

$\frac{f(b) - f(a)}{b - a} = \frac{4 - 19}{3 + 2} = \frac{-15}{5} = -3$ | $f(3) = 18 - 15 + 1 = 4$ | $f(-2) = 8 + 10 + 1 = 19$

29) A rectangle has a perimeter of 100 feet. Find the dimensions of this rectangle that will maximize the area.

A) l = 10, w = 40 B) l = 5, w = 45 C) l = 20, w = 30 D) l = 25, w = 25

$2x + 2y = 100$ | $y = \frac{100 - 2x}{2} = 50 - x$ | $A = x \cdot y$, $A = x(50 - x)$, $A = 50x - x^2$ | $A' = 50 - 2x$, $0 = 50 - 2x$, $2x = 50$ | $x = 25$

30) The function $y = f(x)$ whose first derivative is $y' = (x - 5)(3x - 2)^2$ has inflection point(s) at x = ?

A) 1.5, 1.833 B) $\frac{2}{3}, \frac{28}{9}$ C) $\frac{2}{3}, \frac{32}{9}$ D) 1.833, 5.5

$y'' = (x-5)2(3x-2)\cdot 3 + 1(3x-2)^2 = (6x - 30)(3x - 2) + 9x^2 - 12x + 4$
$= 18x^2 - 102x + 60 + 9x^2 - 12x + 4 = 0$

$27x^2 - 114x + 64$

$x = \frac{114 \pm \sqrt{114^2 - 4 \cdot 27 \cdot 1}}{2(27)}$

$x = 3.5$
$x = .6$

skip

31) A chemical substance is spreading in a nearly circular shape on the surface of the water in a large holding tank pool. At the time the radius of the chemical is increasing at a rate of 1 $\frac{\text{ft}}{\text{minute}}$, the diameter is 10 feet. At what rate is the area of the chemical spreading?

dA/dt $A = \pi r^2$

A) 10 $\frac{\text{ft}^2}{\text{min}}$ B) 31.4159 $\frac{\text{ft}^2}{\text{min}}$ C) 65.7018 $\frac{\text{ft}^2}{\text{min}}$ D) $40\pi \frac{\text{ft}^2}{\text{min}}$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} = 2\pi \cdot 5 \cdot 1$
$= 10\pi$

32) If $\lim_{x \to a} f(x) = L$, where L is a real number, which of the following must be true?

I. $f(a) = L$ II. $\lim_{x \to a^-} f(x) = L$ III. $\lim_{x \to a^+} f(x) = L$

A) I only B) I and II C) II and III D) I, II, and III

33) $\lim_{x \to -\infty} \frac{4x^2 + x - 7}{x^2 - 5x - 3}$ $\frac{4}{1}$

A) 0 B) $\frac{7}{3}$ C) 4 D) Does not exist

34) Which of the following functions are continuous for all real numbers x?

I. $f(x) = |x|$ II. $f(x) = \tan x$ III. $f(x) = 3x^2 + x - 7$

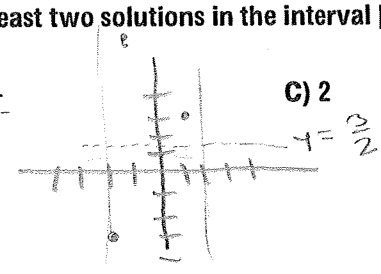
A) II only B) III only C) I and II D) I and III

35) The function f is continuous on the closed interval [-2, 1]. Some values of f are shown in the table.

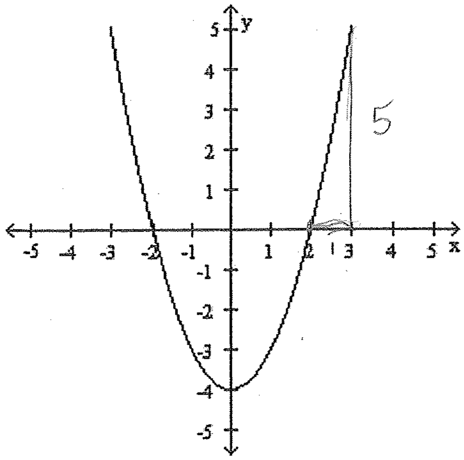
x	-2	-1	0	1
f(x)	-3	7	k	3

The equation $f(x) = \frac{3}{2}$ must have at least two solutions in the interval [-1, 1] if k = ? — K must be below $\frac{3}{2}$

A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$



36) **Consider the function y = f(x) shown below. Approximate the instantaneous rate of change at x = 2.**



A) -4 B) -2 C) 0 D) 4

37) **A function f(x) exists such that $f''(x) = (x - 2)^2(x + 1)$. How many points of inflection does f(x) have?**

A) None B) One C) Two D) Three

f'': $-$ | -1 | $+$ | 2 | $+$

38) **Let g be a function defined and continuous on the closed interval [a, b]. If g has a local minimum at c where $a < c < b$, which of the following statements must be true?**

I. If g ' (c) exists, then g ' (c) = 0. II. g(c) < g (b) III. g is decreasing on [a, b]

A) I only B) II only C) I and II D) I, II, and III

Skip 39) **The diagonal of a square is increasing at a rate of 3 inches per minute. When the area of the square is 18 square inches, how fast (in inches per minute) is the perimeter increasing?**

$\sqrt{18} = s^2$

A) $12\sqrt{2}$ B) $3\sqrt{2}$ C) $\frac{3\sqrt{2}}{2}$ D) $6\sqrt{2}$

$\sqrt{18}^2 + \sqrt{18}^2 = d^2$
$36 = d^2$
$\frac{dd}{dt} = 3$ in/min
$s^2 + s^2 = d^2$
$2s^2 = d^2$
$4s \cdot \frac{ds}{dt} = 2d \cdot \frac{dd}{dt}$

40) **If $xy^2 - y^3 = x^2 - 5$, then $\frac{dy}{dx}$ = ?**

A) $\frac{y^2 - 2x}{3y^2 - 2xy}$ B) $\frac{2x - 5}{2y - 3y^2}$ C) $\frac{2x}{2y - 3y^2}$ D) $\frac{x + y^2}{xy}$

$x \cdot 2y \cdot \frac{dy}{dx} + y^2 - 3y^2 \cdot \frac{dy}{dx} = 2x$ | $\frac{dy}{dx}(2xy - 3y^2) = 2x - y^2$ | $\frac{dy}{dx} = \frac{2x - y^2}{2xy - 3y^2}$

41) **The distance of a particle from its initial position is given by $s(t) = t - 5 + \frac{9}{t+1}$, where s is feet and t is minutes. Find the velocity at t = 1 minute in feet per minute.**

$s'(t) = v(t) = 1 - 9(t+1)^{-2} = 1 - 9(1+1)^{-2} = 1 - \frac{9}{4} = \frac{-5}{4}$

A) $-\frac{5}{4}$ B) $\frac{13}{4}$ C) $-\frac{9}{4}$ D) $-\frac{7}{4}$

42) **Given $f(x) = e^x(\cos x)$, find a value of x whereby $f'(x) = 0$.**

u v

A) 0 B) $\frac{\pi}{4}$ C) $\frac{\pi}{2}$ D) π

$e^x \cdot -\sin x + e^x \cos x = e^x(\cos x - \sin x) = 0$
never = to 0
$\cos x - \sin x = 0$
$\cos x = \sin x$
$x = \frac{\pi}{4}$

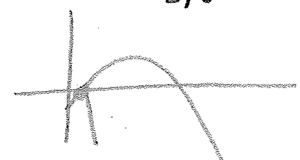
43) $\lim\limits_{x \to 5} \frac{x^2 - 25}{x - 5}$ $= \lim\limits_{x \to 5} \frac{(x+5)\cancel{(x-5)}}{\cancel{x-5}} = x+5 = 5+5 = 10$

A) 0　　B) 5　　C) 10　　D) does not exist

44) If f ' (x) = ln x - x + 2, at which of the following values of x does f have a relative minimum value?

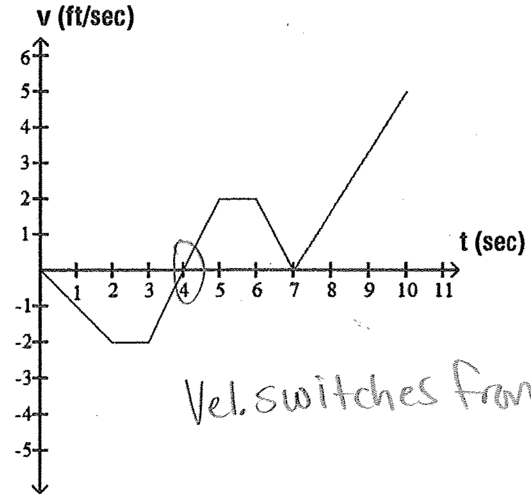
A) 5.146　　B) 3.146　　C) 0.159　　D) 0

Set equal to 0 and graph



X = .1585

45)



Vel. switches from (-) to (+)

When does the body reverse direction?

A) t = 4 sec　　B) t = 2, 3, 5, 6, 7 sec　　C) t = 4 and 7 sec　　D) t = 7 sec