Semester 1 Final Exam

Sem 1 Final Review

Name Key

Form C

Calculators Allowed

True / False: Shade in A for true and B for false

1) The $\lim_{x \to c} f(x)$ can exist even if f(c) is undefined. True

2) If f ' (x) = x + 2, then f(x) has a maximum at x = -2 False

$0 = x + 2 \quad x = -2$

3) If f(x) is a polynomial function, then $\lim_{x \to c} f(x) = f(c)$ always. True

4) The function $f(x) = \frac{1}{x^2}$ is concave up on its entire domain. True

$x^{-2} \quad f' = -2x^{-3} \quad f'' = 6x^{-4} = \frac{6}{x^4}$

$+ \quad 0 \quad +$

5) If f is a function with f ' (5) = 8, then f must be continuous at x = 5.

$m = 8$ True

6) If f(x) = 2x - 3 and g(x) = x + 2, then g(f (4)) = 7. True

$2x - 3 + 2 = 2x - 1 \quad 2 \cdot 4 - 1 = 7$

7) The function f(x) = |x+2| is differentiable everywhere. False

corner

8) The derivative of the product of two functions is equal to the product of the derivative of those functions. False

9) The tangent to the curve $y = \sqrt{x}$ at the point x = 4 crosses the x-axis at the point (-4, 0). True

$y' = \frac{1}{2}x^{-1/2} \quad y = \frac{1}{2\sqrt{4}} = \frac{1}{4} \quad (4, 2)\ m = \frac{1}{4}$

10) The function $y = 5 + e^{2x+1}$ is increasing and concave up everywhere. True

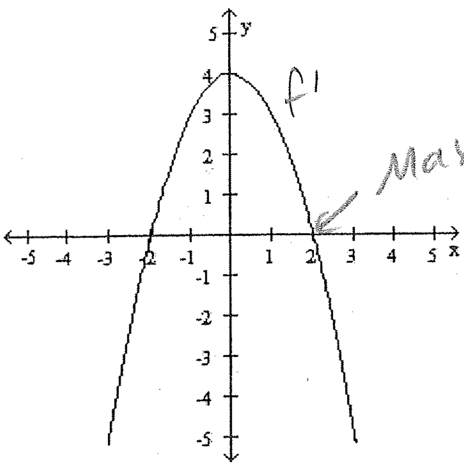
$y - 2 = \frac{1}{4}(x - 4)$

$0 - 2 = \frac{1}{4}(-4 - 4)$

$-2 = \frac{1}{4} \cdot -8$

$-2 = -2$

11) If the following is a graph of f ' (x), then the graph of f(x) has a maximum at (0, 4). False

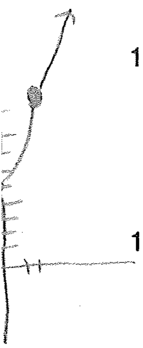


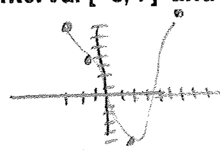
False

Max

f' switches from (+) to (-)

12) If f(x) is a continuous function on the open interval [-3, 7] and f(-1) = 4, f(2) = -5 and f(6) = 8 then f(x) has at least two zeros. True





13) If $f(x) = \begin{cases} x^2 + 5, & \text{if } x < 2 \\ 7x - 5, & \text{if } x \geq 2 \end{cases}$, for all real numbers x, then f(x) is continuous everywhere. True

$7 \cdot 2 - 5 = 9$

$2x = 2(2) = 4 = m$

14) If $f(x) = \begin{cases} x^2 + 5, & \text{if } x < 2 \\ 7x - 5, & \text{if } x \geq 2 \end{cases}$, for all real numbers x, then f(x) is differentiable everywhere. False

$m = 7$

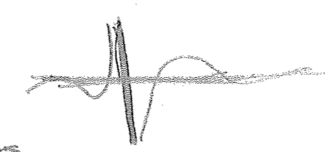
Cusp since $m = 4 \neq 7$

15) If $f(x) = \begin{cases} x^2 + 5, & \text{if } x < 2 \\ 7x - 5, & \text{if } x \geq 2 \end{cases}$, for all real numbers x, then f(x) has a local minimum at x = 2. False

min at $x = 0$

Multiple Choice: Choose the best choice.

16) Find $\lim\limits_{x \to 0} \dfrac{2\sin 3x - 7\cos x}{3x}$ Graph in Radians



A) $\frac{2}{3}$ B) -1.66667 C) does not exist D) 5.372471

17) If $f(x) = \dfrac{2 + x}{3 - x}$, compute f ' (2).

$\dfrac{(3-x)(1) - (2+x)(-1)}{(3-x)^2} = \dfrac{(3-2)(1) + 2 + 2}{(3-2)^2} = \dfrac{5}{1}$

A) 0 B) 2 C) 4 D) 5

18) Find the x- and y- intercepts of the line that is tangent to the curve $y = x^3 - 3x^2$ at the point (-1, -4).

A) (0, 5) and $(-\frac{5}{9}, 0)$ B) (0, -1) and (-4, 0) C) (0, -8) and (2, 0) D) (0, 5) and $(-\frac{6}{5}, 0)$

$y' = 3x^2 - 6x$
$y' = 3x(x-2)$
$= 3(-1)(-1-2)$
$= -3(-3) = 9 = m$

$y + 4 = 9(x + 1)$

$x = 0 \quad y = 5$
$y + 4 = 9$
$y = 5$

$y = 0 \quad x = -5/9$
$0 + 4 = 9x + 9$
$-5 = 9x$
$x = -5/9$

19) The velocity of an object at time t can be described by $v = \frac{t^2 \cos(2t + 3)}{t^2 + 4t + 1}$.

• enter in y =
• 2nd calc, #6, x = 2.75

Use your graphing calculator to approximate the acceleration at time t = 2.75

A) 0.673 B) -1.0027 C) -.057 D) -0.982

20) Find the line that is tangent to the curve $y = \cos\left(\frac{2\pi x}{3}\right) + 1$ at x = 4

$\cos(8\pi/3)$ $y = 1/2$

enter y =, 2nd calc, #6, x = 4, → m = -1.813

A) y = x - 4 B) y = -1.81x - 4 C) y = 4x - 4 D) y = -1.81x + 7.74

$y - \frac{1}{2} = -1.81(x - 4)$

$y = -1.81x + 7.74$

21) Suppose functions f and g and their derivatives have the following values at x = 3

f(3) = 4 f '(3) = $\frac{3}{2}$ g(3) = 5 g '(3) = -3

Find the value of $\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right)$ when x = 3.

$= \frac{f(x) \cdot g'(x) - g(x) \cdot f'(x)}{(f(x))^2} = \frac{4 \cdot -3 - 5 \cdot \frac{3}{2}}{16} = \frac{-12 - \frac{15}{2}}{16} = -\frac{39}{2} \cdot \frac{1}{16}$

A) $-\frac{39}{32}$ B) $-\frac{31}{24}$ C) $-\frac{13}{24}$ D) $\frac{23}{42}$

22) Find the derivative of $y = -2\sin x - 3\sec^2 x$.

$y' = -2\cos - 6\sec x \cdot \sec x \tan x$

A) $\sin x - 3\sec^2 x$ B) $-2\cos x - 6(\sec^2 x)(\tan x)$

C) $2\sin x - 3(\sec^2 x)(\tan x)$ D) $-2\cos x - 6\sec^2 x$

23) The slope of the tangent to $f(x) = \sqrt{3}\cos x$ at $x = \pi$ is

$f' = 0$

A) 3.73 B) 3 C) 0 D) 1.73

24) Find $\frac{dy}{dx}$ if $y = \frac{(3x + 5)^4 - 9x}{5}$. $= \frac{(3x+5)^4}{5} - \frac{9x}{5}$

$f' = \frac{4(3x+5)^3 \cdot 3}{5} - \frac{9}{5}$

A) $12(3x + 5)^3$ B) $12(3x + 5)^3 - 9$ C) $\frac{12(3x + 5)^4 - 9x}{5}$ D) $\frac{12(3x + 5)^3 - 9}{5}$

25) Find an equation for the tangent line to $2x^2 + 2x^2y^3 - y - 10 = 0$ at the point (1, 1).

(1,1)

A) 8x + 13y = 5 B) 8x + 5y = 13 C) 8x - 13y = 5 D) 8x - 5y = 13

$4x + 4x \cdot y^3 + 2x^2 \cdot 3y^2 \cdot \frac{dy}{dx} - 1 \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx}\frac{(2x^2 3y^2 - 1)}{2x^2 3y^2 - 1} = \frac{-4x - 4xy^3}{2x^2 3y^2 - 1}$

26) Determine where $y = 5x^3 + 4x^2 - 12x + 7$ has local maxium or minimum values.

A) max at $x = \frac{2}{3}$, min at $x = -\frac{6}{5}$

B) max at $x = \frac{6}{5}$, min at $x = -\frac{2}{3}$

C) max at $x = -\frac{2}{3}$, min at $x = \frac{6}{5}$

D) max at $x = -\frac{6}{5}$, min at $x = \frac{2}{3}$

Graph calc min/max

x = 1.2 x = .6

$m = \frac{-4 - 4}{6 - 1} = -\frac{8}{5}$

$5\left[y - 1 = -\frac{8}{5}(x - 1)\right]$

← $5y - 5 = -8(x - 1)$

8x + 5y = 13

Calc $\quad min\ x = 1.5\ \ y = 2.975 \qquad max\ x = -1.5\ \ y = 7.025$

27) Find the absolute maximum and the absolute minimum of $y = \frac{2}{5}x^5 - \frac{3}{2}x^3 + 5$ on the interval $[-2, 2]$.

A) max = 11.2667, min = 2.7333
B) max = 7.266, min = -10.831
C) max = 10.754, min = -2.366
D) max = 7.025, min = 2.975

28) Find the value of c that satisfies the Mean Value Theorem for $y = 2x^2 - 5x + 1$ on $-2 \le x \le 3$. $\quad y' = 4x - 5 = -3$

A) $\frac{1}{2}$
B) 2
C) 9
D) $\frac{9}{2}$

$\frac{f(b) - f(a)}{b - a} = \frac{4 - 19}{3 + 2} = \frac{-15}{5} = -3$

$f(3) = 18 - 15 + 1 = 4$

$f(-2) = 8 + 10 + 1 = 19$

$4x = 2$

$x = \frac{1}{2}$

29) A rectangle has a perimeter of 100 feet. Find the dimensions of this rectangle that will maximize the area.

A) l = 10, w = 40
B) l = 5, w = 45
C) l = 20, w = 30
D) l = 25, w = 25

$2x + 2y = 100 \qquad y = \frac{100 - 2x}{2} = 50 - x$

$A = x \cdot y$

$A = x(50 - x)$

$A = 50x - x^2$

$A' = 50 - 2x$

$0 = 50 - 2x$

$2x = 50 \qquad x = 25$

30) The function $y = f(x)$ whose first derivative is $y' = (x - 5)(3x - 2)^2$ has inflection point(s) at $x = $?

A) 1.833, 5.5
B) 1.5, 1.833
C) $\frac{2}{3}, \frac{28}{9}$
D) $\frac{2}{3}, \frac{32}{9}$

$y'' = (x-5)2(3x-2) \cdot 3 + 1(3x-2)^2 = (6x-30)(3x-2) + 9x^2 - 12x + 4$

$= 18x^2 - 102x + 60 + 9x^2 - 12x + 4 = 0$

$27x^2 - 114x + 64 = 0$

$x = \frac{114 \pm \sqrt{114^2 - 4(27)64}}{2(27)}$

$x = 3.5$

$x = .6$

31) A chemical substance is spreading in a nearly circular shape on the surface of the water in a large holding tank pool. At the time the radius of the chemical is increasing at a rate of $1\frac{\text{ft}}{\text{minute}}$, the diameter is 10 feet. At what rate is the area of the chemical spreading? $\quad \frac{dA}{dt} = ?$

A) $31.4159\frac{\text{ft}^2}{\text{min}}$
B) $65.7018\frac{\text{ft}^2}{\text{min}}$
C) $10\frac{\text{ft}^2}{\text{min}}$
D) $40\pi\frac{\text{ft}^2}{\text{min}}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$\frac{dA}{dt} = 2 \cdot \pi (5) 1$

$= 10\pi$

32) If $\lim_{x \to a} f(x) = L$, where L is a real number, which of the following must be true?

I. $f(a) = L$
II. $\lim_{x \to a^-} f(x) = L$
III. $\lim_{x \to a^+} f(x) = L$

A) I only
B) II and III
C) I and II
D) I, II, and III

33) $\lim_{x \to -\infty} \frac{4x^2 + x - 7}{x^2 - 5x - 3}$ $\qquad \frac{4}{1}$

A) Does not exist
B) 0
C) 4
D) $\frac{7}{3}$

34) Which of the following functions are continuous for all real numbers x?

I. $f(x) = |x|$
II. $f(x) = \tan x$
III. $f(x) = 3x^2 + x - 7$

A) I and III
B) I and II
C) II only
D) III only

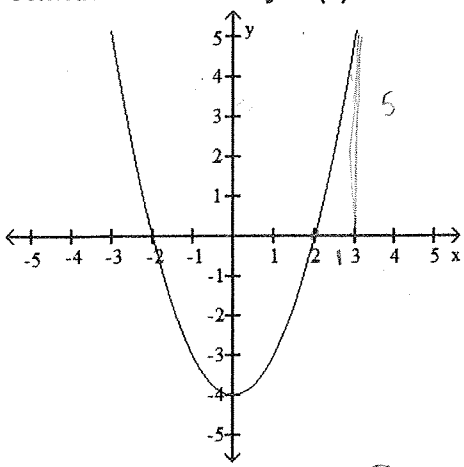
35) The function f is continuous on the closed interval [-2, 1]. Some values of f are shown in the table.

x	-2	-1	0	1
f(x)	-3	7	k	3

The equation $f(x) = \frac{3}{2}$ must have at least two solutions in the interval [-1, 1] if k = ?

A) 2 B) 1 ← Must be below 3/2 C) $\frac{3}{2}$ D) $\frac{5}{2}$

36) Consider the function y = f(x) shown below. Approximate the instantaneous rate of change at x = 2.



$\frac{5}{1}$

A) -2 B) 4 C) -4 D) 0

37) A function f(x) exists such that $f''(x) = (x - 2)^2(x + 1)$. How many points of inflection does f(x) have?

A) One B) Three C) Two D) None

− + +
-1 2

38) Let g be a function defined and continuous on the closed interval [a, b]. If g has a local minimum at c where a < c < b, which of the following statements must be true?

I. If g ' (c) exists, then f ' (c) = 0. II. g(c) < g (b) ~~III.~~ g is decreasing on [a, b]

A) I and II B) I only C) II only D) I, II, and III

$\sqrt{18}^2 + \sqrt{18}^2 = d^2$
$36 = d^2$

$\frac{dd}{dt} = 3$ in/min

$s^2 + s^2 = d^2$
$2s^2 = d^2$

$4s \cdot \frac{ds}{dt} = 2d\frac{dd}{dt}$

$18 = s^2$
$\sqrt{18} = s$

$4 \cdot \sqrt{18} \cdot \frac{ds}{dt} = 2 \cdot 6 \cdot 3$

$\frac{ds}{dt} = \frac{36}{4\sqrt{18}} = 2.1213$

39) The diagonal of a square is increasing at a rate of 3 inches per minute. When the area of the square is 18 square inches, how fast (in inches per minute) is the perimeter increasing? $\frac{dp}{dt} = ?$

$P = 4s$

A) 6 B) $\frac{3\sqrt{2}}{2}$ C) $6\sqrt{2}$ D) $3\sqrt{2}$

$\frac{dp}{dt} = 4 \cdot \frac{ds}{dt}$

$\frac{dp}{dt} = 4 \cdot (2.1213)$

$= 8.485$

40) If $xy^2 - y^3 = x^2 - 5$, then $\frac{dy}{dx} = ?$

A) $\frac{2x - 5}{2y - 3y^2}$ B) $\frac{2x}{2y - 3y^2}$ C) $\frac{x + y^2}{xy}$ D) $\frac{y^2 - 2x}{3y^2 - 2xy}$

$x \cdot 2y \cdot \frac{dy}{dx} + y^2 - 3y^2 \cdot \frac{dy}{dx} = 2x$

$\frac{dy}{dx}(2xy - 3y^2) = 2x - y^2$

$\frac{dy}{dx} = \frac{2x - y^2}{2xy - 3y^2}$

41) The distance of a particle from its initial position is given by $s(t) = t - 5 + \frac{9}{t+1}$, where s is feet and t is minutes.

$v(t) = 1 - 9(t+1)^{-2} \cdot 1$

Find the velocity at t = 1 minute in feet per minute.

$1 - \frac{9}{(1+1)^2} = 1 - \frac{9}{4} = -\frac{5}{4}$

A) $\frac{13}{4}$ B) $\frac{5}{4}$ C) $-\frac{9}{4}$ D) $-\frac{7}{4}$

42) Given $f(x) = e^x (\cos x)$, find a value of x whereby $f'(x) = 0$.

$e^x \cdot -\sin x + e^x \cos x$

$e^x(\cos x - \sin x)$

$e^x = 0$ never

$\cos x = \sin x$

$\frac{\pi}{4}$

A) π B) $\frac{\pi}{2}$ C) 0 D) $\frac{\pi}{4}$

43) $\lim_{x \to 5} \frac{x^2 - 25}{x - 5} = \lim_{x \to 5} \frac{(x+5)\cancel{(x-5)}}{\cancel{x-5}} = 5 + 5 = 10$

A) 5 B) 10 C) 0 D) does not exist

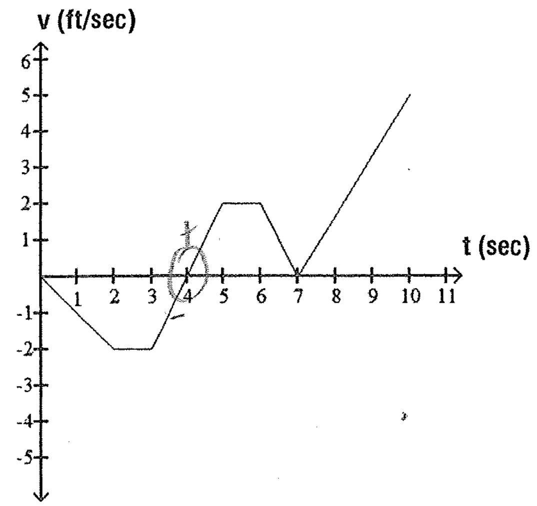
44) If $f'(x) = \ln x - x + 2$, at which of the following values of x does f have a relative minimum value?

A) 3.146 B) 0.159 C) 0 D) 5.146

set equal to zero + graph

$x = .1585$

45)



When does the body reverse direction?

A) t = 4 sec B) t = 2, 3, 5, 6, 7 sec C) t = 7 sec D) t = 4 and 7 sec