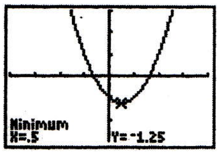
Section 4.3 Exercises

1. $y' = 2x - 1$

Intervals	$x < \frac{1}{2}$	$x > \frac{1}{2}$
Sign of y'	$-$	$+$
Behavior of y	Decreasing	Increasing

Graphical support:



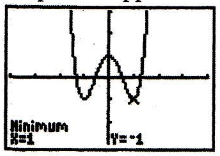
$[-4, 4]$ by $[-3, 3]$

Local (and absolute) minimum at $\left(\frac{1}{2}, -\frac{5}{4}\right)$

3. $y' = 8x^3 - 8x = 8x(x-1)(x+1)$

Intervals	$x < -1$	$-1 < x < 0$	$0 < x < 1$	$1 < x$
Sign of y'	$-$	$+$	$-$	$+$
Behavior of y	Decreasing	Increasing	Decreasing	Increasing

Graphical support:



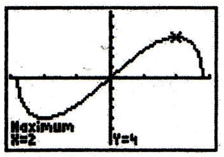
$[-4, 4]$ by $[-3, 3]$

Local maximum: $(0, 1)$;
local (and absolute) minima: $(-1, -1)$ and $(1, -1)$

5. $y' = x\frac{1}{2\sqrt{8-x^2}}(-2x) + (\sqrt{8-x^2}\,(1) = \frac{8-2x^2}{\sqrt{8-x^2}}$

Intervals	$-\sqrt{8} < x < -2$	$-2 < x < 2$	$2 < x < \sqrt{8}$
Sign of y'	$-$	$+$	$-$
Behavior of y	Decreasing	Increasing	Decreasing

Graphical support:



$[-3.02, 3.02]$ by $[-6.5, 6.5]$

Local maxima: $(-\sqrt{8}, 0)$ and $(2, 4)$;

local minima: $(-2, -4)$ and $(\sqrt{8}, 0)$

Note that the local extrema at $x = \pm 2$ are also absolute extrema.

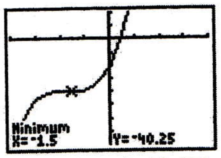
7. $y' = 12x^2 + 42x + 36 = 6(x+2)(2x+3)$

Intervals	$x < -2$	$-2 < x < -\frac{3}{2}$	$-\frac{3}{2} < x$
Sign of y'	$+$	$-$	$+$
Behavior of y	Increasing	Decreasing	Increasing

$y'' = 24x + 42 = 6(4x+7)$

Intervals	$x < -\frac{7}{4}$	$-\frac{7}{4} < x$
Sign of y'	$-$	$+$
Behavior of y	Concave down	Concave up

Graphical support:



$[-4, 4]$ by $[-80, 20]$

(a) $\left(-\frac{7}{4}, \infty\right)$

(b) $\left(-\infty, -\frac{7}{4}\right)$

9. $y' = \frac{2}{5}x^{-4/5}$

Intervals	$x < 0$	$0 < x$
Sign of y'	$+$	$+$
Behavior of y	Increasing	Increasing

$y'' = -\frac{8}{25}x^{-9/5}$

Intervals	$x < 0$	$0 < x$
Sign of y''	$+$	$-$
Behavior of y	Concave up	Concave down

Graphical support:

$[-6, 6]$ by $[-1.5, 7.5]$

(a) $(-\infty, \infty)$

(b) None

(c) $(-\infty, 0)$

(d) $(0, \infty)$

(e) None

(f) $(0, 3)$

13. $y = xe^x$

$y' = e^x + xe^x$

Intervals	$x < -1$	$x > -1$
Sign of y'	$-$	$+$
Behavior of y	Decreasing	Increasing

$y'' = 2e^x + xe^x$

Intervals	$x < -2$	$x > -2$
Sign of y''	$-$	$+$
Behavior of y	Concave down	Concave up

$\left(-2, -\frac{2}{e^2}\right)$

15. $y' = \frac{1}{1+x^2}$

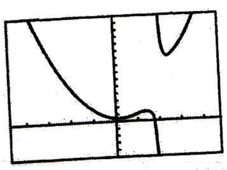
since $y' > 0$ for all x, y is always increasing:

$$y'' = \frac{d}{dx}(1+x^2)^{-1} = -(1+x^2)^{-2}(2x) = \frac{-2x}{(1+x^2)^2}$$

Intervals	$x < 0$	$0 < x$
Sign of y''	$+$	$-$
Behavior of y	Concave up	Concave down

(0, 0)

19. We use a combination of analytic and grapher techniques to solve this problem. Depending on the viewing window chosen, graphs obtained using NDER may exhibit strange behavior near $x = 2$ because, for example, NDER $(y, 2) \approx 1{,}000{,}000$ while y' is actually undefined at $x = 2$. The graph of $y = \frac{x^3 - 2x^2 + x - 1}{x - 2}$ is shown below.

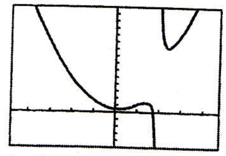


[−4,7, 4.7] by [−5, 15]

19. Continued

$$y' = \frac{(x-2)(3x^2-4x+1)-(x^3-2x^2+x-1)(1)}{(x-2)^2}$$
$$= \frac{2x^3-8x^2+8x-1}{(x-2)^2}$$

The graph of y' is shown below.



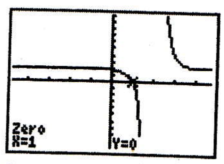
[−4,7, 4.7] by [−10, 10]

The zeros of y' are $x \approx 0.15$, $x \approx 1.40$, and $x \approx 2.45$.

Intervals	$x < 0.15$	$0.15 < x < 1.40$	$1.40 < x < 2$	$2 < x < 2.45$	$2.45 < x$
Sign of y'	$-$	$+$	$-$	$-$	$+$
Behavior of y	Decreasing	Increasing	Decreasing	Decreasing	Increasing

$$y'' = \frac{(x-2)^2(6x^2-16x+8)-(2x^3-8x^2+8x-1)(2)(x-2)}{(x-2)^4}$$
$$= \frac{(x-2)(6x^2-16x+8)-2(2x^3-8x^2+8x-1)}{(x-2)^3}$$
$$= \frac{2x^3-12x^2+24x-14}{(x-2)^3}$$
$$= \frac{2(x-1)(x^2-5x+7)}{(x-2)^3}$$

The graph of y'' is shown below.



[−4,7, 4.7] by [−10, 10]

Note that the discriminant of $x^2 - 5x + 7$ is $(-5)^2 - 4(1)(7) = -3$, so the only solution of $y'' = 0$ is $x = 1$.

Intervals	$x < 1$	$1 < x < 2$	$2 < x$
Sign of y''	$+$	$-$	$+$
Behavior of y	Concave up	Concave down	Concave up

(1, 1)

21. (a) Zero: $x = \pm 1$;
positive: $(-\infty, -1)$ and $(1, \infty)$;
negative: $(-1, 1)$

(b) Zero: $x = 0$;
positive: $(0, \infty)$;
negative: $(-\infty, 0)$

23. **(a)** $(-\infty, -2]$ and $[0, 2]$

(b) $[-2, 0]$ and $[2, \infty)$

(c) Local maxima: $x = -2$ and $x = 2$;
local minimum: $x = 0$

25. **(a)** $v(t) = x'(t) = 2t - 4$

(b) $a(t) = v'(t) = 2$

(c) It begins at position 3 moving in a negative direction. It moves to position -1 when $t = 2$, and then changes direction, moving in a positive direction thereafter.

29. **(a)** The velocity is zero when the tangent line is horizontal, at approximately $t = 2.2$, $t = 6$ and $t = 9.8$.

(b) The acceleration is zero at the inflection points, approximately $t = 4$, $t = 8$ and $t = 11$.

33. $y = 3x - x^3 + 5$

$y' = 3 - 3x^2$
$y'' = -6x$
$y' = 0$ at ± 1.
$y''(-1) > 0$ and $y''(1) < 0$, so there is a local minimum at $(-1, 3)$ and a local maximum at $(1, 7)$.

37. $y = xe^x$

$y' = (x+1)e^x$
$y'' = (x+2)e^x$
$y' = 0$ at -1.
$y''(-1) > 0$, so there is a local minimum at $(-1, -1/e)$.

39. $y' = (x-1)^2(x-2)$

Intervals	$x < 1$	$1 < x < 2$	$2 < x$
Sign of y'	−	−	+
Behavior of y	Decreasing	Decreasing	Increasing

$$y'' = (x-1)^2(1) + (x-2)(2)(x-1) = (x-1)[(x-1) + 2(x-2)] = (x-1)(3x-5)$$

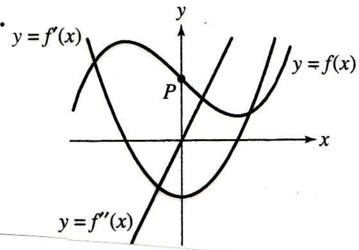
Intervals	$x < 1$	$1 < x < \frac{5}{3}$	$\frac{5}{3} < x$
Sign of y''	+	−	+
Behavior of y	Concave up	Concave down	Concave up

(a) There are no local maxima.

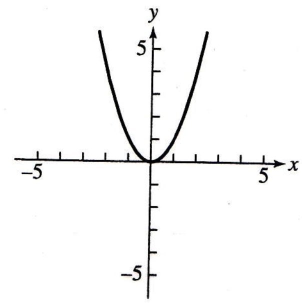
(b) There is a local (and absolute) minimum at $x = 2$.

(c) There are points of inflection at $x = 1$ and at $x = \frac{5}{3}$.

41.



45. One possible answer:



49. **(a)** $[0, 1]$, $[3, 4]$, and $[5.5, 6]$

(b) $[1, 3]$ and $[4, 5.5]$

(c) Local maxima: $x = 1$, $x = 4$
(if f is continuous at $x = 4$), and $x = 6$;
local minima: $x = 0$, $x = 3$, and $x = 5.5$

51. **(a)** Absolute maximum at $(1, 2)$;
absolute minimum at $(3, -2)$

(b) None

(c) One possible answer: