AB Calculus
6.4 Practice worksheet

Name Key

(1) Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is $\frac{dy}{dt} = ky$, where y is the amount of oil left in the well at any time t. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

a) Write an equation for y, the amount of oil remaining in the well at any time t.
b) At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?
c) In order not to lose money, at what time t should oil no longer be pumped from the well?

a) $\frac{dy}{dt} = Ky$

$\int \frac{1}{y}dy = \int K dt$

$\ln|y| = Kt + C$

$e^{Kt+C} = y$

$e^{Kt} \cdot e^{C} = y$, $e^C = y_0$

$y = 1{,}000{,}000e^{Kt}$

$500{,}000 = 1{,}000{,}000\, e^{K \cdot 6}$

$\frac{1}{2} = e^{6K}$

$\ln\frac{1}{2} = 6K$

$K = \frac{\ln .5}{6}$

$K = -.115524\ldots$

$\boxed{y = 1{,}000{,}000\, e^{-.1155\ldots t}}$

b) $\frac{dy}{dt} = -.1155\ldots(y)$

$= -.1155\ldots(600{,}000)$

$= \boxed{-69{,}314.718 \text{ gal/yr}}$

c) $50{,}000 = 1{,}000{,}000e^{-.1155t}$

$\frac{1}{20} = e^{-.1155\ldots t}$

$\frac{\ln 1/20}{-.1155\ldots} = t$

$\boxed{t = 25.932 \text{ years}}$

(2) A yam is put in a 200°C oven and heats up according to the differential equation

$\frac{dH}{dt} = -k(H - 200)$, with k as a positive constant.

(a) If the yam is at 20°C when it is put into the oven, solve the differential equation.

(b) Find k using the fact that after 30 minutes the temperature of the yam is 120°C.

A) $\int \frac{dH}{H-200} = \int -k \cdot dt$

$\ln(H-200) = -kt + C$

$e^{-kt+C} = H - 200$

$e^{-kt} \cdot e^{C} = H - 200$

$Ae^{-kt} = H - 200$

$H = 200 + Ae^{-kt}$

$20 = 200 + Ae^{0}$

$A = -180$

$\boxed{H = -180e^{-kt} + 200}$

B) $120 = -180e^{-k \cdot 30} + 200$

$-80 = -180e^{-30k}$

$\frac{-80}{-180} = \frac{-180}{-180}$

$\frac{4}{9} = e^{-30k}$

$\ln\frac{4}{9} = -30k$

$k = \frac{\ln\frac{4}{9}}{-30}$

$\boxed{k \approx .027}$

(3) The number of bacteria in a culture is growing at a rate of $1500e^{3t/4}$ per unit of time, t. At t=0, then number of bacteria present was 2,000. Find the number present at t=4.

(A) $2000e^3$ (B) $6000e^3$ (C) $2000e^6$

(D) $1500e^6$ (E) $1500e^3 + 500$

$\int e^{3t/4} dt$, $u = 3t/4$, $du = \frac{3}{4}dt$, $\frac{4}{3}du = dt$

$\frac{4}{3}e^{u} + C$

Start → $\frac{db}{dt} = 1500e^{3t/4}$

$db = 1500e^{3t/4}$ → $\int db = \int 1500e^{3t/4}$

$b = 1500e^{3t/4} \cdot \frac{4}{3} + C = 2000e^{3t/4} + C = B$ → $2000 = 2000e^{3 \cdot 0/4} + C$

$t=0, B=2000$; $0 = C$

$B = 2000e^{\frac{3 \cdot 4}{4}} = 2000e^{3}$ (A)

(4) The change in N, the number of bacteria in a culture dish at time t, is given by $\frac{dN}{dt} = 2N$. If N=3 when t=0, the approximate value of t when N=1210 is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

$\frac{dN}{dt} = 2N$

$\int \frac{1}{N}dN = 2\int dt$

$\ln N = 2t + C$

$e^{2t+C} = N$

$e^{2t} \cdot e^{C} = N$

$t=0, N=3$

$e^{2 \cdot 0} \cdot e^{C} = 3$

$e^{C} = 3$

$3e^{2t} = N$

$3e^{2t} = 1210$

$e^{2t} = 403\frac{1}{3}$

$\ln e^{2t} = \ln 403\frac{1}{3}$

$t = \frac{\ln 403\frac{1}{3}}{2} = 2.999$