AP Calculus

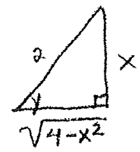
Name Answer Key

Date

Derivatives of Inverse Trig. Functions

Directions: Find the first derivative of the inverse trigonometric functions.

1. $y = \sin^{-1}\left(\frac{x}{2}\right)$



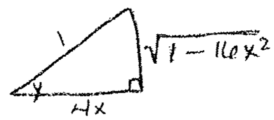
$\sin y = \frac{x}{2}$

$\cos y \frac{dy}{dx} = \frac{1}{2}$

$\frac{dy}{dx} = \frac{1}{2\cos y}$

$\frac{dy}{dx} = \frac{1}{\sqrt{4-x^2}}$

2. $y = \cos^{-1}(4x)$



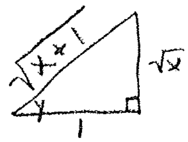
$\cos y = 4x$

$-\sin y \frac{dy}{dx} = 4$

$\frac{dy}{dx} = \frac{4}{-\sin y}$

$\frac{dy}{dx} = \frac{4}{-\sqrt{1-16x^2}}$

3. $y = \tan^{-1}\sqrt{x}$



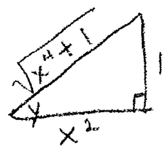
$\tan y = \sqrt{x}$

$\sec^2 y \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}\,\sec^2 y}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}\,(x+1)}$

4. $y = \cot^{-1}x^2$



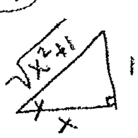
$\cot y = x^2$

$-\csc^2 y \frac{dy}{dx} = 2x$

$\frac{dy}{dx} = \frac{2x}{-\csc^2 y}$

$\frac{dy}{dx} = \frac{2x}{-(x^4+1)}$

5. $y = \tan^{-1}\left(\frac{1}{x}\right)$



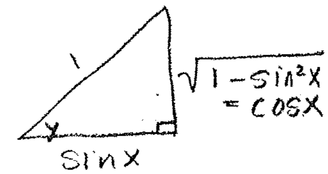
$\tan y = \frac{1}{x}$

$\sec^2 y \frac{dy}{dx} = -\frac{1}{x^2}$

$\frac{dy}{dx} = \frac{-1}{x^2\sec^2 y}$

$\frac{dy}{dx} = \frac{-1}{x^2+1}$

6. $y = \cos^{-1}(\sin x)$



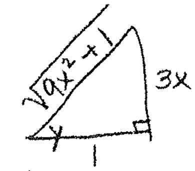
$\cos y = \sin x$

$-\sin y \frac{dy}{dx} = \cos x$

$\frac{dy}{dx} = \frac{\cos x}{-\sin y}$

$\frac{dy}{dx} = \frac{\cos x}{-\cos x} = -1$

7. $y = 5\tan^{-1}3x$



$y' = 5$ (Deriv. of $\tan^{-1}(3x)$)

$y' = \frac{15}{9x^2+1}$

$\tan y = 3x$

$\sec^2 y \frac{dy}{dx} = 3$

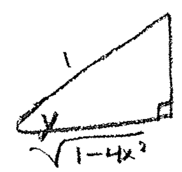
$\frac{dy}{dx} = \frac{3}{\sec^2 y}$

$\frac{dy}{dx} = \frac{3}{9x^2+1}$

8. $y = (\sin^{-1}2x)^3$

$y' = 3(\sin^{-1}2x)^2$ (Deriv. of $\sin^{-1}(2x)$)

$y' = \frac{6(\sin^{-1}2x)^2}{\sqrt{1-4x^2}}$



$\sin y = 2x$

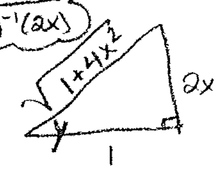
$\cos y \frac{dy}{dx} = 2$

$\frac{dy}{dx} = \frac{2}{\cos y} = \frac{2}{\sqrt{1-4x^2}}$

9. $y = \sqrt{\tan^{-1}(2x)}$

$y' = \frac{1}{2}(\tan^{-1}(2x))^{-\frac{1}{2}}$ (Deriv. of $\tan^{-1}(2x)$)

$y' = \frac{1}{(1+4x^2)\sqrt{\tan^{-1}(2x)}}$



$\tan y = 2x$

$\sec^2 y \frac{dy}{dx} = 2$

$\frac{dy}{dx} = \frac{2}{\sec^2 y} = \frac{2}{1+4x^2}$

10. $y = (\csc^{-1}(x^2+1))^6$

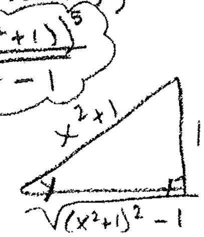
$y' = 6(\csc^{-1}(x^2+1))^5$ (Deriv. of $\csc^{-1}(x^2+1)$)

$y' = \frac{12x(\csc^{-1}(x^2+1))^5}{-(x^2+1)\sqrt{(x^2+1)^2-1}}$

$\csc y = x^2+1$

$-\csc y \cot y \frac{dy}{dx} = 2x$

$\frac{dy}{dx} = \frac{2x}{-\csc y \cot y} = \frac{2x}{-(x^2+1)\sqrt{(x^2+1)^2-1}}$



AP Calculus

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Derivatives of Inverse Functions

Directions: Show your work neatly on a separate sheet of paper.

A. Find the derivative of f^{-1} for each of the following functions:

1. $f(x) = 5x^3 + x - 7$

$x = 5y^3 + y - 7$

$1 = 15y^2 \frac{dy}{dx} + \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{15y^2+1}$

2. $f(x) = 2x^5 + x^3 + 1$

$x = 2y^5 + y^3 + 1$

$1 = 10y^4 \frac{dy}{dx} + 3y^2 \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{10y^4+3y^2}$

3. $f(x) = 5x - \sin(2x)$

$x = 5y - \sin(2y)$

$1 = 5\frac{dy}{dx} - 2\cos(2y)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{5-2\cos(2y)}$

B. Evaluating the Derivatives of Inverse Functions

1. Find the derivative of the inverse of $f(x) = x^3 + 7x + 2$ at the point where $f^{-1}(10) = 1$.

$x = y^3 + 7y + 2$

$1 = 3y^2\frac{dy}{dx} + 7\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{3y^2+7}$

$\frac{dy}{dx} = \frac{1}{10}$

2. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

$x = y^3 + y$

$1 = 3y^2\frac{dy}{dx} + \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{3y^2+1}$

$\frac{dy}{dx} = \frac{1}{4}$

3. Let f be the function defined by $f(x) = x^3 + 8x + \cos(3x)$. If $g(x) = f^{-1}(x)$ and $g(1) = 0$, find the value of $g'(1)$.

$x = y^3 + 8y + \cos(3y)$

$1 = 3y^2\frac{dy}{dx} + 8\frac{dy}{dx} - 3\sin(3y)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{3y^2+8-3\sin(3y)}$

$\frac{dy}{dx} = \frac{1}{8}$

4. If $f(x) = x^5 + 3x + 2$ and $g(x) = f^{-1}(x)$, find $g'(2)$.

$x = y^5 + 3y + 2$

$1 = 5y^4\frac{dy}{dx} + 3\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{5y^4+3}$

$\frac{dy}{dx} = \frac{1}{3}$

$g(2) = ?$

$f(?) = 2$

$g(2) = 0$

$2 = x^5 + 3x + 2$

$0 = x^5 + 3x$

$0 = x(x^4 + 3)$

$x = 0$ never $= 0$

5. Find $(f^{-1})'(-1)$ if $f(x) = 3x - \cos x$.

$x = 3y - \cos y$

$1 = 3\frac{dy}{dx} + \sin y\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{3+\sin y}$

$\frac{dy}{dx} = \frac{1}{3}$

$f^{-1}(-1) = ?$

$f(?) = -1$

$f^{-1}(-1) = 0$

$3x - \cos x = -1$

$0 - \cos 0 = -1$

$-1 = -1$

6. Find $(f^{-1})'(5)$ if $f(x) = x^3 + 2x + 5$.

$x = y^3 + 2y + 5$

$1 = 3y^2\frac{dy}{dx} + 2\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{3y^2+2}$

$\frac{dy}{dx} = \frac{1}{2}$

$f^{-1}(5) = ?$

$f(?) = 5$

$f^{-1}(5) = 0$

$x^3 + 2x + 5 = 5$

$x^3 + 2x = 0$

$x(x^2 + 2) = 0$