Name Key

1. Suppose $\int_0^1 f(x)dx = 2$, $\int_1^2 f(x)dx = 3$, $\int_0^1 g(x)dx = -1$, and $\int_0^2 g(x)dx = 4$. Compute the following using the properties of the integral.

(a) $\int_1^2 g(x)dx = \int_0^2 g(x)dx - \int_0^1 g(x)dx = 4 - -1 = 5$

(b) $\int_0^2 [2f(x) - 3g(x)]dx = 2\int_0^2 f(x)dx - 3\int_0^2 g(x)dx = 2\left(\int_0^1 f(x) + \int_1^2 f(x)\right) - 3\left(\int_0^2 g(x)\right)$
$= 2(2+3) - 3(4) = 10 - 12 = -2$

(c) $\int_1^1 g(x)dx = \boxed{0}$

(d) $\int_1^2 f(x)dx + \int_2^0 g(x)dx = 3 + -\int_0^2 g(x) = 3 + -4 = -1$

(e) $\int_0^2 f(x)dx + \int_2^1 g(x)dx$

$\int_0^1 f(x) + \int_1^2 f(x) + -\int_1^2 g(x) = 2 + 3 + -\left(\int_0^2 g(x) - \int_0^1 g(x)\right)$
$= 5 + -(4 - -1) = 5 + -(5) = \boxed{0}$

For 2-7, interpret the integrand as the rate of change of a quantity and evaluate the integral using the antiderivative of the quantity, as in Example 4.

2. $\int_0^{\pi/2} \cos x\, dx = \sin x \Big|_0^{\pi/2}$
$= \sin\frac{\pi}{2} - \sin 0$
$= 1 - 0 = 1$

3. $\int_0^{1/2} \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x \Big|_0^{1/2}$
$= \sin^{-1}\frac{1}{2} - \sin^{-1} 0$
$= \frac{\pi}{6} - 0 = \boxed{\frac{\pi}{6}}$

4. $\int_0^{\pi/4} \sec^2 x\, dx$
$= \tan x \Big|_0^{\pi/4} = \tan\frac{\pi}{4} - \tan 0$
$= 1 - 0 = 1$

5. $\int_3^7 8\, dx$
$8x \Big|_3^7 = 56 - 24 = \boxed{32}$

6. $\int_1^4 -x^{-2}\, dx$
$= \frac{-x^{-1}}{-1}\Big|_1^4 = \frac{1}{x}\Big|_1^4 = \frac{1}{4} - 1 = \frac{3}{4}$

7. $\int_{-1}^2 3x^2\, dx = \frac{3x^3}{3}\Big|_{-1}^2 = x^3\Big|_{-1}^2$
$= 2^3 - (-1)^3 = 8+1 = 9$

For 8 + 9, find the average value of the function on the interval, using antiderivatives to compute the integral.

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

8. $y = \frac{1}{x}$, $[e, 2e]$ (a, b)

x^{-1}

$$\frac{1}{2e-e}\int_e^{2e} \frac{1}{x}\,dx = \frac{1}{e}\cdot \ln|x| \Big|_e^{2e} = \frac{1}{e}\ln|2e| - \frac{1}{e}\ln|e|$$

$$\boxed{\frac{1}{e}\ln\left|\frac{2e}{e}\right|} = \boxed{\frac{1}{e}\ln|2|}$$

9. $y = \frac{1}{1+x^2}$, $[0, 1]$ (a, b)

$$\frac{1}{1-0}\int_0^1 \frac{1}{1+x^2}\,dx = \tan x \Big|_0^1 = \tan 1 - \tan 0$$

$$= \tan 1 - 0$$

$$\boxed{= \tan 1 \approx 1.557}$$