AP Calculus 3.7 WKST

Name Key

Date

Implicit Differentiation Practice

Directions: Show all work to support your answer. Evaluate WITHOUT a calculator!

1. Given $x^4 + xy = 7$, find $\frac{dy}{dx}$ and then evaluate $\frac{dy}{dx}$ at the point $(-1,2)$.

$4x^3 + x\cdot\frac{dy}{dx} + 1\cdot y = 0$

$x\frac{dy}{dx} = -y - 4x^3$

$\boxed{\frac{dy}{dx} = \frac{-y-4x^3}{x}}$

$\frac{-(2)-4(-1)^3}{-1} = \frac{-2+4}{-1} = \frac{2}{-1} = \boxed{-2}$

2. Given $2xy - y^2 = 3y^4$, find $\frac{dy}{dx}$ and then evaluate $\frac{dy}{dx}$ at the point $(3,1)$.

$2x\cdot\frac{dy}{dx} + 2y - 2y\frac{dy}{dx} = 12y^3\frac{dy}{dx}$

$2x\frac{dy}{dx} - 2y\frac{dy}{dx} - 12y^3\frac{dy}{dx} = -2y$

$\frac{dy}{dx}(2x - 2y - 12y^3) = -2y$

$\boxed{\frac{dy}{dx} = \frac{-2y}{2x-2y-12y^3}}$

$= \frac{-2(1)}{2(3)-2(1)-12(1)}$

$= \frac{-2}{6-2-12} = \frac{-2}{-8} = \boxed{\frac{1}{4}}$

3. Given $3x^2 - xy^3 = 6x$, find $\frac{dy}{dx}$ and then evaluate $\frac{dy}{dx}$ at the point $(-2,-1)$.

$6x - (1y^3 + x\cdot 3y^2\cdot\frac{dy}{dx}) = 6$

$-x3y^2\frac{dy}{dx} = 6 - 6x + y^3$

$\boxed{\frac{dy}{dx} = \frac{6-6x+y^3}{-x\cdot 3y^2}}$

$\frac{dy}{dx} = \frac{6-6(-2)+(-1)^3}{-(-2)\cdot 3(-1)^2} = \frac{6+12-1}{2\cdot 3}$

$= \boxed{\frac{17}{6}}$

4. Given $x^2 + y^2 = 3$, find $\frac{d^2y}{dx^2}$.

$2x + 2y\cdot\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

$\frac{d^2y}{dx^2} = \frac{y\cdot -1 - \frac{dy}{dx}\cdot(-x)}{y^2} = \frac{-y + \frac{dy}{dx}\cdot x}{y^2}$

$= \frac{-y + \frac{-x}{y}\cdot x}{y^2} = \boxed{\frac{-y + \frac{-x^2}{y}}{y^2}}$