

Differentiation - Chain Rule

Differentiate each function with respect to x .

1) $y = (x^3 + 3)^5$

$$\begin{aligned}\frac{dy}{dx} &= 5(x^3 + 3)^4 \cdot 3x^2 \\ &= 15x^2(x^3 + 3)^4\end{aligned}$$

2) $y = (-3x^5 + 1)^3$

$$\begin{aligned}\frac{dy}{dx} &= 3(-3x^5 + 1)^2 \cdot -15x^4 \\ &= -45x^4(-3x^5 + 1)^2\end{aligned}$$

3) $y = (-5x^3 - 3)^3$

$$\begin{aligned}\frac{dy}{dx} &= 3(-5x^3 - 3)^2 \cdot -15x^2 \\ &= -45x^2(-5x^3 - 3)^2\end{aligned}$$

4) $y = (5x^2 + 3)^4$

$$\begin{aligned}\frac{dy}{dx} &= 4(5x^2 + 3)^3 \cdot 10x \\ &= 40x(5x^2 + 3)^3\end{aligned}$$

5) $f(x) = \sqrt[4]{-3x^4 - 2}$

$$\begin{aligned}f'(x) &= \frac{1}{4}(-3x^4 - 2)^{-\frac{3}{4}} \cdot -12x^3 \\ &= -\frac{3x^3}{(-3x^4 - 2)^{\frac{3}{4}}}\end{aligned}$$

6) $f(x) = \sqrt{-2x^2 + 1}$

$$\begin{aligned}f'(x) &= \frac{1}{2}(-2x^2 + 1)^{-\frac{1}{2}} \cdot -4x \\ &= -\frac{2x}{(-2x^2 + 1)^{\frac{1}{2}}}\end{aligned}$$

7) $f(x) = \sqrt[3]{-2x^4 + 5}$

$$\begin{aligned}f'(x) &= \frac{1}{3}(-2x^4 + 5)^{-\frac{2}{3}} \cdot -8x^3 \\ &= -\frac{8x^3}{3(-2x^4 + 5)^{\frac{2}{3}}}\end{aligned}$$

8) $y = (-x^4 - 3)^{-2}$

$$\begin{aligned}\frac{dy}{dx} &= -2(-x^4 - 3)^{-3} \cdot -4x^3 \\ &= \frac{8x^3}{(-x^4 - 3)^3}\end{aligned}$$

$$9) y = (3x^3 + 1)(-4x^2 - 3)^4$$

$$\begin{aligned} \frac{dy}{dx} &= (3x^3 + 1) \cdot 4(-4x^2 - 3)^3 \cdot -8x + (-4x^2 - 3)^4 \cdot 9x^2 \\ &= x(-4x^2 - 3)^3(-132x^3 - 32 - 27x) \end{aligned}$$

$$10) y = \frac{(x^3 + 4)^5}{3x^4 - 2}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(3x^4 - 2) \cdot 5(x^3 + 4)^4 \cdot 3x^2 - (x^3 + 4)^5 \cdot 12x^3}{(3x^4 - 2)^2} \\ &= \frac{3x^2(x^3 + 4)^4(11x^4 - 10 - 16x)}{(3x^4 - 2)^2} \end{aligned}$$

$$11) y = ((x + 5)^5 - 1)^4$$

$$\begin{aligned} \frac{dy}{dx} &= 4((x + 5)^5 - 1)^3 \cdot 5(x + 5)^4 \\ &= 20((x + 5)^5 - 1)^3 \cdot (x + 5)^4 \end{aligned}$$

$$12) y = (5x^3 - 3)^5 \sqrt[4]{-4x^5 - 3}$$

$$\begin{aligned} \frac{dy}{dx} &= (5x^3 - 3)^5 \cdot \frac{1}{4}(-4x^5 - 3)^{-\frac{3}{4}} \cdot -20x^4 + (-4x^5 - 3)^{\frac{1}{4}} \cdot 5(5x^3 - 3)^4 \cdot 15x^2 \\ &= \frac{5x^2(5x^3 - 3)^4(-65x^5 - 45 + 3x^2)}{(-4x^5 - 3)^{\frac{3}{4}}} \end{aligned}$$

Critical thinking question:

13) Give a function that requires three applications of the chain rule to differentiate. Then differentiate the function.

Many answers: Ex $y = (((2x + 1)^5 + 2)^6 + 3)^7$

$$\frac{dy}{dx} = 7(((2x + 1)^5 + 2)^6 + 3)^6 \cdot 6((2x + 1)^5 + 2)^5 \cdot 5(2x + 1)^4 \cdot 2$$