AP Calculus (-1,0) (0,1) 3.6 WKST #2 Name Key
Date

Chain Rule with Trig. Functions Practice

Directions: Find the derivative of $f(x)$, then evaluate the derivative at the given x-value. No calculator!

1. If $f(x) = \overset{u}{x^3} \overset{v}{\cos x}$, find $f'(\pi)$

$f'(x) = -x^3 \cdot \sin x + 3x^2 \cos x$

$f'(\pi) = -(\pi)^3 \cdot \sin\pi + 3(\pi)^2 \cos\pi$

$= -\pi^3 \cdot 0 + 3\pi^2 \cdot -1 = \boxed{-3\pi^2}$

2. If $f(x) = \overset{u}{\sqrt{x}} \overset{v}{\sin x}$, find $f'(2\pi)$

$f'(x) = \sqrt{x} \cdot \cos x + \frac{1}{2} x^{-\frac{1}{2}} \sin x$

$= \sqrt{x} \cos x + \frac{1}{2\sqrt{x}} \sin x$

$f'(2\pi) = \sqrt{2\pi} \cdot \cos 2\pi + \frac{1}{2\sqrt{2\pi}} \cdot \cancel{\sin 2\pi}_{0}$

$= -\sqrt{2\pi}$

3. If $f(x) = \frac{\sec x}{x}$, find $f'(x)$

$f'(x) = \frac{x \cdot \sec x \tan x - 1 \cdot \sec x}{x^2}$

4. If $f(x) = \frac{\tan x}{x^2}$, find $f'(x)$

$f'(x) = \frac{x^2 \cdot \sec^2 x - 2x \tan x}{x^4}$

5. If $f(x) = \sin(2x)$, find $f'\left(\frac{\pi}{2}\right)$

$f'(x) = \cos(2x) \cdot 2 = 2\cos(2x)$

$f'\left(\frac{\pi}{2}\right) = 2 \cdot \cos\left(2 \cdot \frac{\pi}{2}\right) = 2 \cdot \cos\pi = 2 \cdot -1$

$= -2$

6. If $f(x) = \cos\left(\frac{x}{2}\right)$, find $f'\left(\frac{\pi}{2}\right)$

$f'(x) = -\frac{1}{2} \sin \frac{x}{2}$

$f'\left(\frac{\pi}{2}\right) = -\frac{1}{2} \sin\left(\frac{\pi}{2} \cdot \frac{1}{2}\right) = -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{-\frac{\sqrt{2}}{4}}$

7. If $f(x) = \sec(x^2)$, find $f'(\sqrt{\pi})$

$f'(x) = \sec(x^2)(\tan x^2) \cdot 2x$

$f'(\sqrt{\pi}) = \sec(\sqrt{\pi}^2) \tan(\sqrt{\pi}^2) \cdot 2(\sqrt{\pi})$

$= -1 \cdot 0 \cdot 2\sqrt{\pi} = \boxed{0}$

8. If $f(x) = \tan^2 x$, find $f'\left(\frac{\pi}{4}\right)$

$f'(x) = 2\tan x \cdot \sec^2 x$

$f'\left(\frac{\pi}{4}\right) = 2\tan\frac{\pi}{4} \cdot \sec^2\left(\frac{\pi}{4}\right) = 2 \cdot 1 \cdot \frac{2}{\sqrt{2}} \cdot \frac{2}{\sqrt{2}} =$

$= \frac{8}{2} = 4$

9. If $f(x) = \csc^5(x)$, find $f'\left(\frac{\pi}{2}\right)$

$f'(x) = 5\csc^4 x \cdot -\cot x \csc x$

$= -5\csc^5 x \cdot \cot x$

$f'\left(\frac{\pi}{2}\right) = -5\csc^5\left(\frac{\pi}{2}\right) \cdot \underset{=0}{\cot\left(\frac{\pi}{2}\right)} = \boxed{0}$

10. If $f(x) = \cos^3(4x)$, find $f'\left(\frac{\pi}{16}\right)$

$f'(x) = 3\cos^2(4x) \cdot -\sin(4x) \cdot 4$

$= -12\cos^2(4x)\sin(4x)$

$f'\left(\frac{\pi}{16}\right) = -12\cos^2\left(4 \cdot \frac{\pi}{16}\right)\sin\left(4 \cdot \frac{\pi}{16}\right)$

$= -12\cos^2\left(\frac{\pi}{4}\right) \cdot \sin\left(\frac{\pi}{4}\right) = -12 \cdot \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{-3\sqrt{2}}$

11. If $f(x) = \cot^3(2x)$, find $f'\left(\frac{\pi}{12}\right)$

$f'(x) = 3\cot^2(2x) \cdot -\csc^2(2x) \cdot 2$

$= -6\cot^2(2x)\csc^2(2x)$

$f'\left(\frac{\pi}{12}\right) = -6\cot^2\left(\frac{\pi}{6}\right)\csc^2\left(\frac{\pi}{6}\right)$

$= -6 \cdot \sqrt{3}\sqrt{3} \cdot 2 \cdot 2 = \boxed{-72}$

12. If $f(x) = \sin^3(3x^2)$, find $f'\left(\frac{\sqrt{\pi}}{3}\right)$

$f'(x) = 3\sin^2(3x^2)\cos(3x^2) \cdot 6x$

$= 18x \sin^2(3x^2)\cos(3x^2)$

$f'\left(\frac{\sqrt{\pi}}{3}\right) = 18 \cdot \frac{\sqrt{\pi}}{3} \cdot \sin^2\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)$

$= 18 \cdot \frac{\sqrt{\pi}}{3} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{9\sqrt{\pi}}{4}}$