3.4 skip 3b, #3

AP CALCULUS
VELOCITY & SPEED PRACTICE

Name Key

Directions: No calculator! Use your notes!

1) A particle moves along a coordinate line in such a way that is position is given by $s(t) = 2\sqrt{t}(2 - 2t + t^2)$ for $t \geq 0$. For what times is the particle moving to the left?

A) $0 < t < 2$
B) The particle never moves to the left.
C) $1 < t < 2$
D) $2 < t < 5$
E) $t > 5$

$2t^{\frac{1}{2}}$

vel (−)

$v(t) = 2\sqrt{t}(-2+2t) + \frac{1}{\sqrt{t}}(2-2t+t^2)$

$v(t) = \frac{1}{\sqrt{t}}(2t(-2+2t) + 2 - 2t + t^2)$

$v(t) = \frac{1}{\sqrt{t}}(-4t + 4t^2 + 2 - 2t + t^2)$

$v(t) = \frac{1}{\sqrt{t}}(5t^2 - 6t + 2)$

$\frac{1}{\sqrt{t}} = 0$ Never

$5t^2 - 6t + 2 = 0$ Never $(b^2 - 4ac = -4)$

Never crosses x-axis

0 | + → Always moving right

$t \geq 0$

2) A particle moves along the x-axis so that its position at time t is given by $x(t) = 2t^2 - 12t + 9$. For what value of t is the particle at rest?

A) 1
B) 4
C) 3
D) 0
E) 9

$v(t) = 4t - 12 = 0$

$4t = 12$

$t = 3$

3) ~~X~~ A particle moves along the x-axis in such a way that its position is given by $x(t) = 2t^3 + 24t - 4\cos\left(\frac{\pi}{2}t\right)$ for $t > 0$. When is the particle moving to the left?

A) $t > 0$
B) $0 < t < 2$
C) The particle never moves to the left.
D) $0 < t < \frac{\pi}{2}$
E) $t > \frac{\pi}{2}$

skip trig

4) A particle moves along a horizontal axis so that its position is given by $x(t) = 4t^5 - 5t^3$ for any time t. How many times does the particle change direction?

A) 1
B) 2
C) 3
D) 0
E) 5

$v(t) = 20t^4 - 15t^2$

$0 = 5t^2(4t^2 - 3)$

$5t^2 = 0$ $\quad 4t^2 - 3 = 0$

$t = 0$ $\quad 4t^2 = 3$

$t^2 = \frac{3}{4}$

$t = \pm\frac{\sqrt{3}}{2}$

− | + | − | +
$-\frac{\sqrt{3}}{2}$ 0 $\frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} \approx .866$

5) A particle moves along the x-axis so that its position at any time t is given by $x(t) = t^4 - 6t^2 - 3$. Which of the following *best* describes the motion of the particle for $0 < t < 1$?

A) Moving to the right and speeding up.
B) Moving to the left and speeding up.
C) Moving to the right and slowing down.
D) Moving to the left at a constant speed.
E) Moving to the left and slowing down.

$v(t) = 4t^3 - 12t = 0$

$4t(t^2 - 3) = 0$

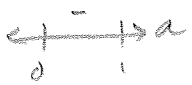
$t = 0 \quad t^2 = 3$

$t = \pm\sqrt{3} \approx 1.73$ not in interval

$a(t) = 12t^2 - 12 = 0$

$= 12(t^2 - 1) = 0$

$t = \pm 1$



Velocity is (-) and decreasing
... speeding up, moving left

7) A particle moves along the x-axis so that is position at any time t is given by $x(t) = t^3 - 6t^2 + 9t + 12$. During what times is the speed of the particle increasing?

A) $t < 1$ or $t > 3$
B) $1 < t < 2$ or $t > 3$
C) $t < 1$ or $2 < t < 3$
D) $t < 2$ or $t > 3$
E) $1 < t < 3$

$v(t) = 3t^2 - 12t + 9 = 0$

$3(t^2 - 4t + 3) = 0$

$3(t - 3)(t - 1) = 0$

$t = 3 \quad t = 1$

$a(t) = 6t - 12 = 0$

$6t = 12$

$t = 2$

Speed ↑
vel. +, incr
vel. -, decr

~~6)~~ A particle moves along the x-axis so that its position is given by $x(t) = 2\cos(2t)$ for $0 \le t \le \pi$. Which statement *best* describes the motion of the particle for $\frac{\pi}{4} < t < \frac{\pi}{2}$?

A) moving to the right at increasing speed
B) moving to the left at decreasing speed
C) moving to the left at constant speed
D) moving to the right at decreasing speed
E) moving to the left at increasing speed

Skip

trig

8) A particle moves along the x-axis so that its position is given by $x(t) = 4t^3 - 3t^2$ for any time $t \ge 0$. During what time interval is the particle's position to the left ~~of zero?~~

A) $0 < t < \frac{3}{4}$
B) $0 < t < \frac{4}{3}$
C) $\frac{1}{2} < t < \frac{3}{4}$
D) $1 < t < \frac{3}{4}$
E) $0 < t < \frac{1}{2}$

$v(t) = 12t^2 - 6t = 0$

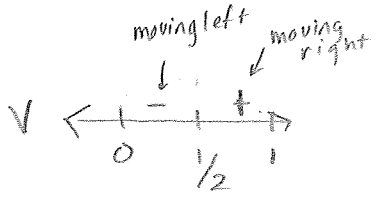
$6t(2t - 1) = 0$

$t = 0 \quad t = \frac{1}{2}$

$a(t) = 24t - 6 = 0$

$24t = 6$

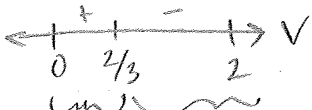
$t = \frac{1}{4}$



9) A particle moves on a straight line in such a way that its distance at any time t from a fixed point on the line is given by $S(t) = 4t - 3t^2$. What is the total distance traveled by the particle between $t = 0$ and $t = 2$?

A) $\frac{16}{3}$
B) $\frac{20}{3}$
C) $\frac{22}{3}$
D) 6
E) 4

$V(t) = 4 - 6t = 0$
$4 = 6t$
$\frac{2}{3} = t$



$\frac{4}{3}$ $\left|-4 - \frac{4}{3}\right| = \left|-\frac{16}{3}\right| = \frac{16}{3}$

$S(0) = 0$
$S(\frac{2}{3}) = \frac{4}{3}$
$S(2) = -4$

Total Distance $= \frac{4}{3} + \frac{16}{3} = \frac{20}{3}$

10. A particle moves along the x-axis in such a way that its position at time t for $t \geq 0$ is given by $x(t) = \frac{1}{3}t^3 - 3t^2 + 8t$.

A. When is the particle at rest? when $V = 0$
B. When is the particle moving to the right? when $V(+)$
C. Find all values of t for which the particle moving to the left. $V(-)$
D. When does the particle change direction?
E. What is the velocity of the particle at $t = 3$?
F. What is the total distance that the particle traveled from time $t = 0$ to $t = 3$?

A] $t^2 - 6t + 8 = 0$
$(t-2)(t-4) = 0$
$t = 2, t = 4$

B] 

$0 < t < 2$
$t > 4$

C] $2 < t < 4$

D] $t = 2, 4$

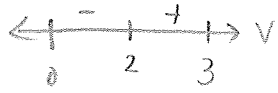
E] $3^2 - 6(3) + 8 = 9 - 18 + 8 = -1$

F] $x(0) = 0$ $x(2) = 20/3$ $x(3) = 6$ $\frac{20}{3} + \left|6 - \frac{20}{3}\right| = \frac{20}{3} + \frac{2}{3} = \frac{22}{3}$

11. A particle moves along a line such that its position function is given by $s(t) = t^3 - 12t + 3$ for the time interval [0, 3]. Find the following:

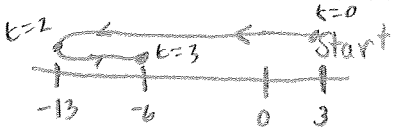
A. When the particle is at rest, moving to the left, and to the right.
B. The intervals where the particle is speeding up and slowing down.
C. Draw a picture that illustrates the motion of the particle.
D. The total distance that the particle travels over the indicated time interval.

$V(t) = 3t^2 - 12 = 0$
$= 3(t^2 - 4) = 0$
$(t+2)(t-2) = 0$
$t = -2, 2$



A] At rest, $t = 2$
Left: $0 < t < 2$
Right: $2 < t < 3$

C] $S(0) = 3$
$S(2) = -13$
$S(3) = -6$



$a(t) = 6t = 0$
$t = 6$

B] Slowing down: $(0,2) \leftarrow t$
Speeding up: $(2,3) \leftarrow t$

D] $|3 - -13| + |-6 - -13|$
$16 + 7 = 23$ units