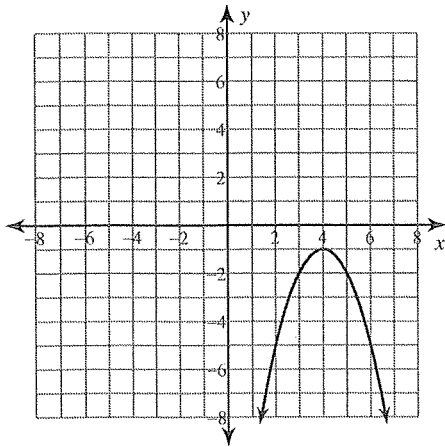


Mean Value Theorem

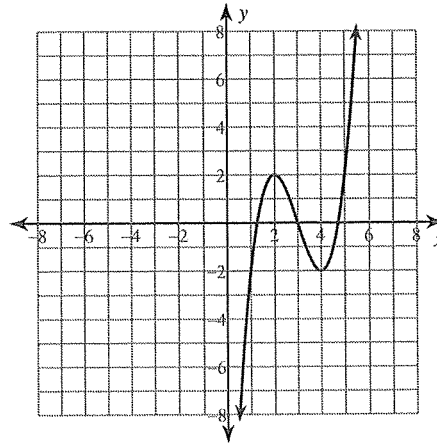
For each problem, find the values of c that satisfy the Mean Value Theorem.

1) $y = -x^2 + 8x - 17$; $[3, 6]$



$$\left\{ \frac{9}{2} \right\}$$

2) $y = x^3 - 9x^2 + 24x - 18$; $[2, 4]$



$$\left\{ \frac{9 + \sqrt{3}}{3}, \frac{9 - \sqrt{3}}{3} \right\}$$

3) $y = -\frac{x^2}{2} + x - \frac{1}{2}$; $[-2, 1]$

$$\left\{ -\frac{1}{2} \right\}$$

4) $y = \frac{x^2}{2} - 2x - 1$; $[-1, 1]$

$$\{0\}$$

5) $y = x^3 + 3x^2 - 2$; $[-2, 0]$

$$\left\{ \frac{-3 + \sqrt{3}}{3}, \frac{-3 - \sqrt{3}}{3} \right\}$$

6) $y = -x^3 + 4x^2 - 3$; $[0, 4]$

$$\left\{ \frac{8}{3} \right\}$$

7) $y = \frac{x^2 - 9}{3x}$; $[1, 4]$

$$\{2\}$$

8) $y = \frac{x^2}{2x - 4}$; $[-4, 1]$

$$\{2 - \sqrt{6}\}$$

$$9) y = -(-2x + 6)^{\frac{1}{2}}; [-2, 3]$$

$$\left\{ \frac{7}{4} \right\}$$

$$10) y = -(-5x + 25)^{\frac{1}{2}}; [3, 5]$$

$$\left\{ \frac{9}{2} \right\}$$

For each problem, determine if the Mean Value Theorem can be applied. If it can, find all values of c that satisfy the theorem. If it cannot, explain why not.

step 11) $y = -\frac{x^2}{4x + 8}; [-3, -1]$

The function is not continuous on $[-3, -1]$

12) $y = \frac{-x^2 + 9}{4x}; [1, 3]$

$$\{\sqrt{3}\}$$

step 13) $y = -(6x + 24)^{\frac{2}{3}}; [-4, -1]$

$$\left\{ -\frac{28}{9} \right\}$$

14) $y = (x - 3)^{\frac{2}{3}}; [1, 4]$

The function is not differentiable on $(1, 4)$

Critical thinking question:

step 15) Use the Mean Value Theorem to prove that $|\sin a - \sin b| \leq |a - b|$ for all real values of a and b where $a \neq b$.

Let $f(x) = \sin x$. Use the interval $[a, b]$. By the MVT, we know that there is at least one c

such that $\frac{\sin b - \sin a}{b - a} = \cos c$. We know $\cos c \leq 1$ for all c . Therefore, $\frac{\sin b - \sin a}{b - a} \leq 1$,

$$\frac{|\sin a - \sin b|}{|a - b|} \leq 1, \text{ and } |\sin a - \sin b| \leq |a - b|.$$