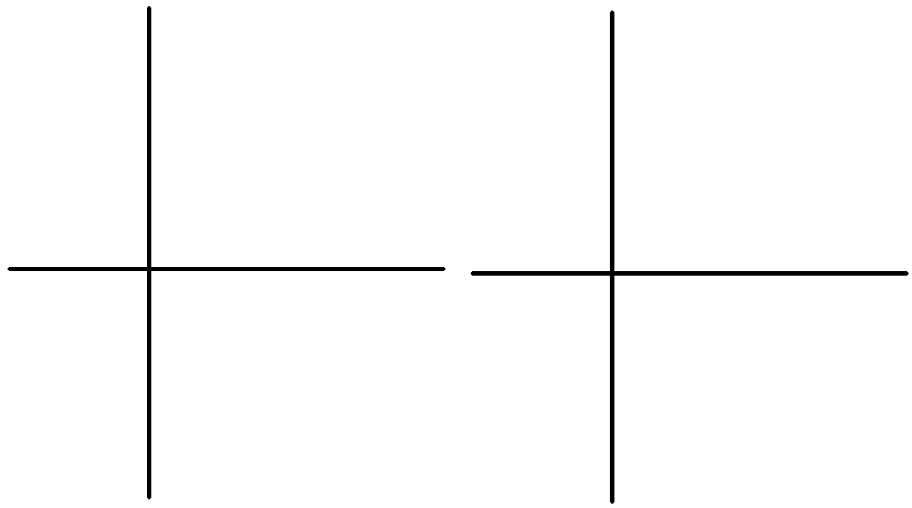
2.3 Continuity - (Continuous)

Can you trace the graph without lifting your pencil?



Interior Points

A function f(x) is continuous at an interior point if $\lim_{x \to c} f(x) = f(c)$

Endpoints

A function f(x) is continuous at the left endpoint if $\lim_{x \to a+} f(x) = f(a)$

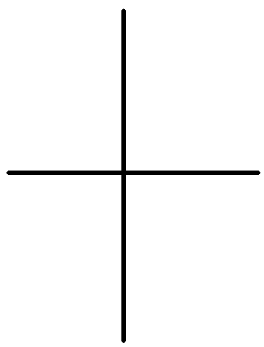
A function f(x) is continuous at the right endpoint if $\lim_{x \to a-} f(x) = f(a)$

4 types of discontinuity

1. Removable (Hole in the graph)

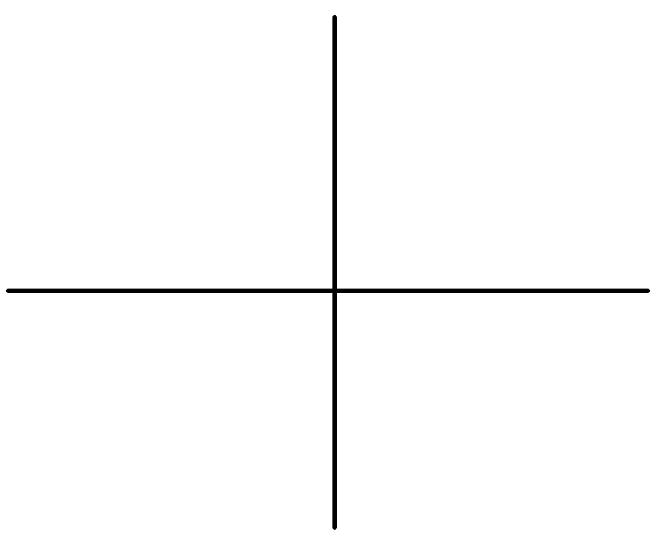
- Can be removed by filling in the missing point

Ex 1) $f(x) = \frac{x^2 - 1}{x - 1}$



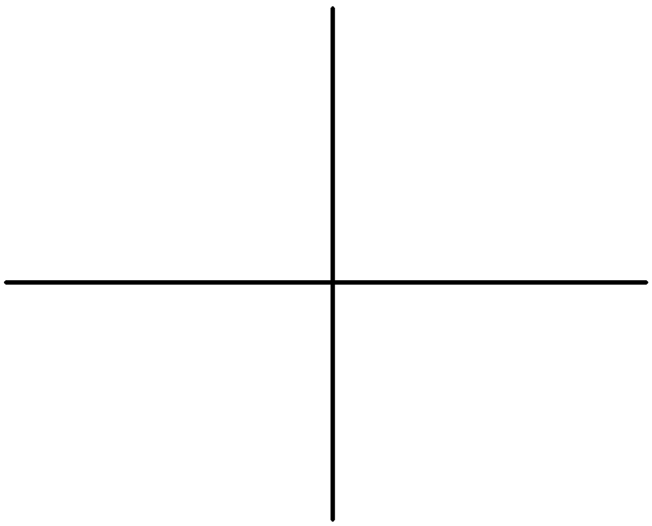
2. Infinite (Vertical Asymptote)

Ex 2) $f(x) = \frac{x^2 + 2x + 1}{x - 1}$



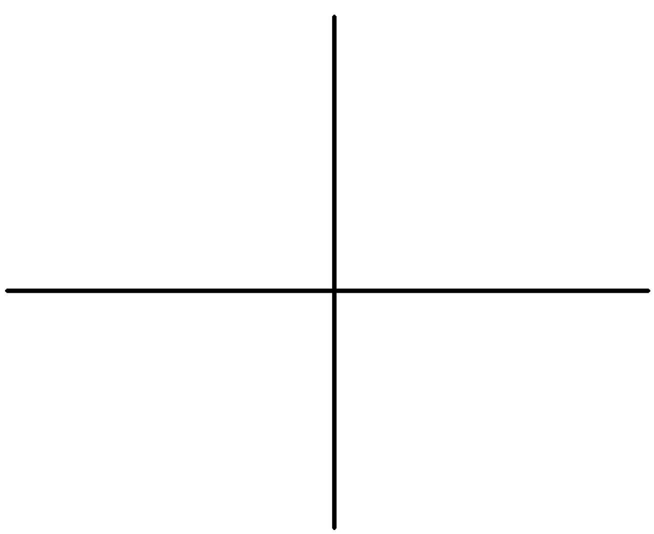
3. Jump (Piecewise Functions)

Ex 3) $f(x) = \begin{cases} x + 1, & x > 0 \\ x^2, & x \leq 0 \end{cases}$



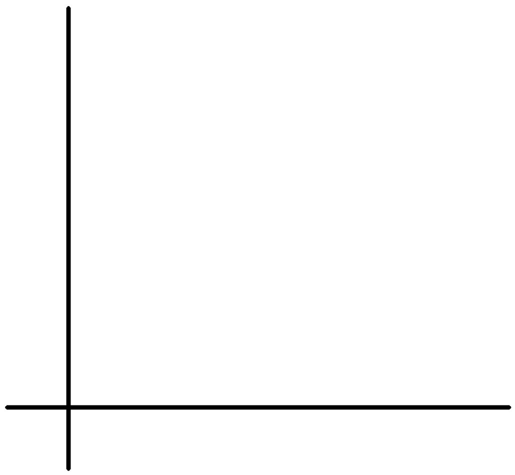
4. Oscillating

Ex 4) $f(x) = \sin(1/x)$



Intermediate Value Theorem

If a function is continuous on the interval [a,b], then f(x) must take on all y-values between f(a) and f(b)



Ex 5) The function f is continuous on the closed interval [1, 3] and has the value given in the table. The equation f(x) = (5/4) must have at least two solutions in the interval [1, 3] if k =

x	1	2	3
f(x)	2	k	4

A) 1/4

B) 3/2

C) 2

D) 9/4

E) 3

Ex 6) Let f be a continuous function on the closed interval [-2, 5] . If f(-2) = 3 and f(5) = -7, then the Intermediate Value Theorem guarantees that

A) $-7 \le f(x) \le 3$ for all x between -2 and 5.

B) f(c) = -3 for at least one c between -2 and 5.

C) f(c) = 0 for at least one c between -7 and 3.

D) f(x) is continually decreasing between -2 and 5.