2.1 Rates of Change and Limits Day 2

Let us take a look at what happens to this function when using substitution to find the limit.

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

So, let us try finding the limit another way using factoring.

Ex 1) $$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

Ex 2) $$\lim_{x \to 1} \frac{t^3 - t}{t - 3}$$

Ex 3) $$\lim_{h \to 0} \frac{(h - 5)^2 - 25}{h}$$

One-sided limits

Right hand limits

$$\lim_{x \to c^+} f(x)$$

limit as x approaches c from the right

Left hand limits

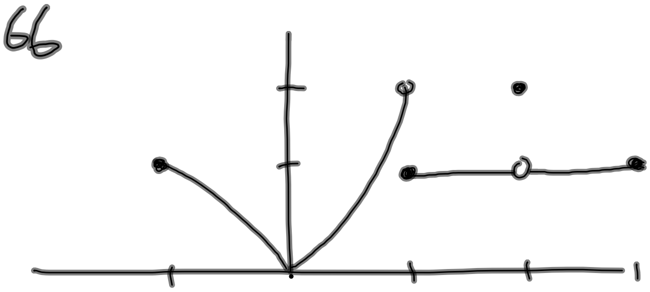
$$\lim_{x \to c^-} f(x)$$

limit as x approaches c from the left

*A function f(x) has a limit if and only if the left and right hand limits are equal.

$$\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = \lim_{x \to c} f(x) = L$$

Ex 4) #38 p.66



True or False?

a) $\lim_{x\to -1^+} f(x) = 1$

b) $\lim_{x\to 2} f(x) = \text{DNE}$

c) $\lim_{x\to 2} f(x) = 2$

d) $\lim_{x\to 1^-} f(x) = 2$

e) $\lim_{x\to 1^+} f(x) = 1$

f) $\lim_{x\to 1} f(x) = \text{DNE}$

g) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$

h) $\lim_{x\to c} f(x)$ exists at every c in (-1,1)

i) $\lim_{x\to c} f(x)$ exists at every c in (1,3).

Sandwich / Squeeze Theorem

If $g(x) \leqslant f(x) \leqslant h(x)$
for values of c in an open interval and

$$\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L$$

THEN

$$\lim_{x\to c} h(x) = L$$

Ex 5) $\lim_{x \to 0} x \sin \frac{1}{x}$