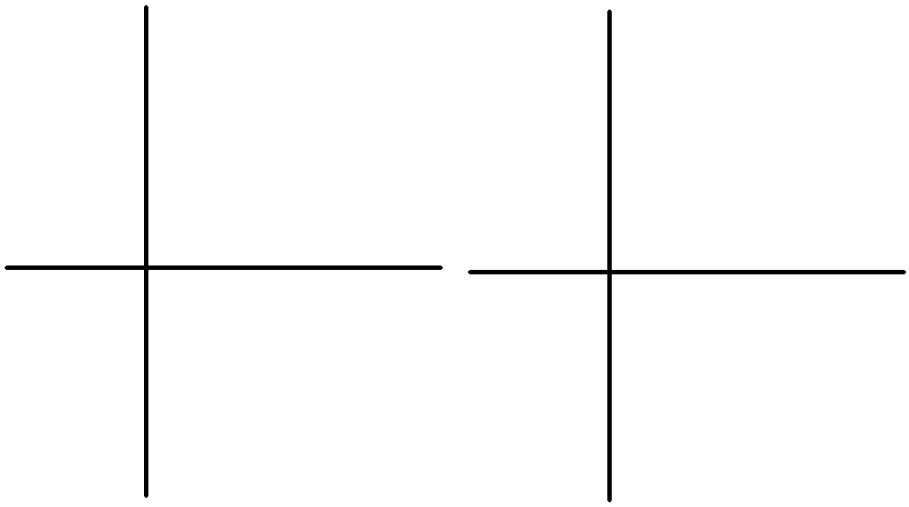
2.3 Continuity - (Continuous)

Can you trace the graph without lifting your pencil?



Interior Points

A function f(x) is continuous at an interior point if $\lim_{x \to c} f(x) = f(c)$

Endpoints

A function f(x) is continuous at the left endpoint if $\lim_{x \to a+} f(x) = f(a)$

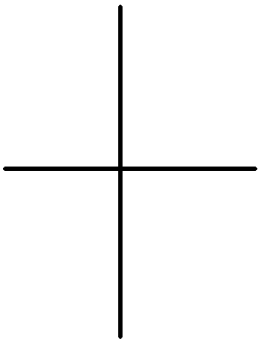
A function f(x) is continuous at the right endpoint if $\lim_{x \to a-} f(x) = f(a)$

4 types of discontinuity

1. Removable (Hole in the graph)

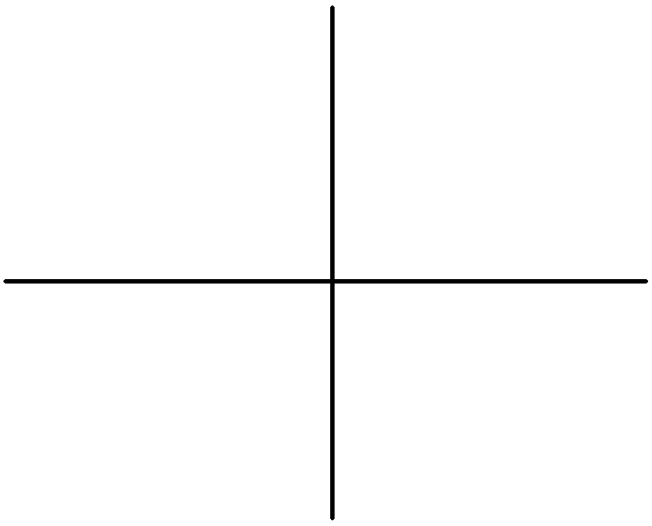
   - Can be removed by filling in the missing point

Ex 1) $f(x) = \frac{x^2 - 1}{x - 1}$



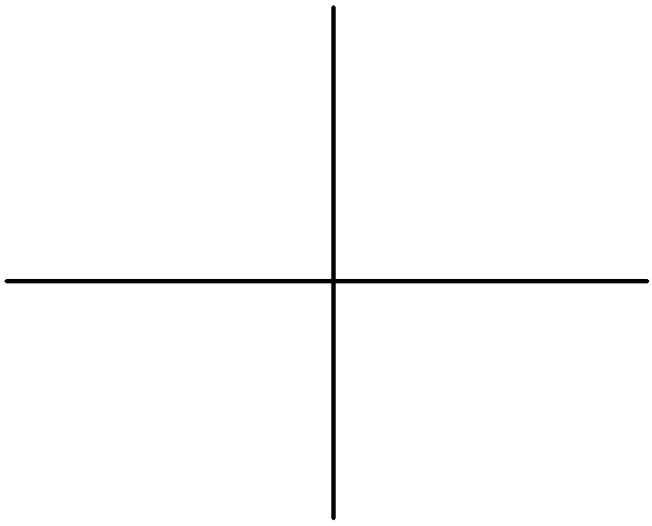
2. Infinite (Vertical Asymptote)

Ex 2) $f(x) = \frac{x^2 + 2x + 1}{x - 1}$



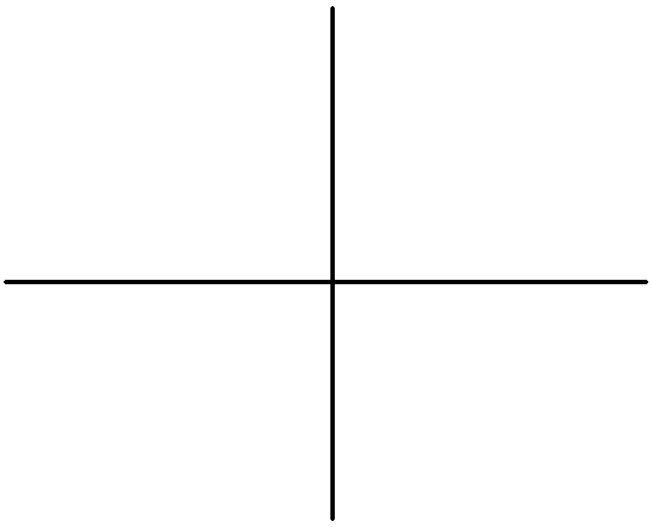
3. Jump (Piecewise Functions)

Ex 3) $f(x) = \begin{cases} x + 1, & x > 0 \\ x^2, & x \leq 0 \end{cases}$



4. Oscillating

Ex 4) $f(x) = \sin(1/x)$



Intermediate Value Theorem

If a function is continuous on the interval [a,b], then f(x) must take on all y-values between f(a) and f(b)