

4.1 Extreme Values of Functions  
Day 2

**Extreme Value Theorem**

If  $f$  is continuous on a closed interval  $[a,b]$ , then  $f$  has both a maximum and minimum value.

Ex 1)

$$y = x^2 \quad 0 \leq x \leq 2$$

$$y = x^2 \quad 0 \leq x < 2$$

A local maximum value occurs if and only if  $f(x) \leq f(c)$  for all  $x$  in an interval.

A local minimum value occurs if and only if  $f(x) \geq f(c)$  for all  $x$  in an interval.

Ex 2) Find the extreme values of the function.

$$f(x) = \begin{cases} 5 - 2x^2, & x \leq 1 \\ x + 2, & x > 1 \end{cases}$$

True or False

If  $f(c)$  is a local max of a continuous function  $f$  on an open interval  $(a,b)$ , then  $f'(c) = 0$

If  $m$  is a local minimum and  $M$  is a local maximum of a continuous function  $f$  on  $(a,b)$ , then  $m < M$ .

Ex 3)  $f(x) = 4x - x^2 + 6$      $[0,4]$

Find the absolute maximum value.

Ex 4) If  $f$  is a continuous, decreasing function on  $[0,10]$  with a critical point at  $(4,2)$ , which of the following statements must be false?

- a.  $f(10)$  is an absolute minimum of  $f$  on  $[0,10]$
- b.  $f(4)$  is neither a relative max nor min
- c.  $f'(4)$  does not exist
- d.  $f'(4) = 0$
- e.  $f'(4) < 0$

Ex 5) Which of the following functions has exactly two local extrema on its domain?

a.  $y = |x-2|$

b.  $y = x^3 - 6x + 5$

c.  $y = x^3 + 6x - 5$

d.  $y = \tan x$

e.  $y = x + \ln x$

Ex 6) If an even function  $f$  with domain all real numbers has a local maximum at  $x = a$ , then  $f(-a)$ ...

a. is a local minimum

b. is a local maximum

c. is both a local minimum and a local max

d. could be either a local min or local max

e. is neither a local min nor a local max