5.3 Definite Integrals and Antiderivatives

Rules for Definite Integrals

A specific starting and stopping point

$$\int_a^b f(x)\,dx$$

Zero Property

$$\int_a^a f(x)\,dx = 0$$

Constant Multiple Rule

$$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_b^c f(x)\,dx + \int_a^b f(x)\,dx = \int_a^c f(x)\,dx$$

$$\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Ex 1)

Given

$$\int_1^9 f(x)\,dx = -1$$

$$\int_7^9 f(x)\,dx = 5$$

$$\int_7^9 h(x)\,dx = 4$$

$$\int_7^9 f(x) + h(x) =$$

$$\int_9^1 f(x)\,dx =$$

$$\int_7^9 3h(x)\,dx =$$

$$\int_1^7 f(x)\,dx =$$

$$\int_9^7 h(x) - f(x) =$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

F(x) is the anti-derivative of f(x)

f(x) is the derivative of F(x)

Ex 2)

$f(x) = 2x$	$F(x) =$
$f(x) = \cos x$	$F(x) =$
$f(x) = -\sin x$	$F(x) =$
$f(x) = e^x$	$F(x) =$
$f(x) = 1/x$	$F(x) =$
$f(x) = 4$	$F(x) =$

Ex 3) $\int_{1}^{7} 4 \, dx$

Ex 4) $\int_{0}^{2\pi} \cos x \, dx$

Ex 5) $\int_{1}^{4} 2x \, dx$

Ex 6) $\int_{-2}^{5} 3x^2 \, dx$

Average Value (y-value)

$$= \frac{1}{b-a}\left(\int_a^b f(x)dx\right)$$

Ex 7) Find the average value of $f(x) = 3x^2 - 1$ on $[0, 4]$.